EASTERN UNIVERSITY, SRI LANKA

FACULTY OF SCIENCE

FIRST YEAR SECOND SEMESTER EXAMINATION IN SCIENCE

2021/2022 (Aug./Sep. 2024)

MT 1242 - MATHEMATICAL MODELING I

Answer all questions **Time: 02 hours**

Q1. (a) Match the items in Column A with the items in Column B by choosing the most suitable unique answer for each item.

A	*B*
Parameter sensitivity	Long-term behaviour
Future prediction	Modeling principles
Distribution of possible outcomes	Cause-effect diagram
Parameter reduction	Non-dimensionalisation
Causal relationship	Effect on solution
Newtonian planetry motion	Conservation of resources
Animals' hierarchy	Stochastic model
Quaries and description	Deterministic model
Per-capita growth rate	Mechanistic model
Flow diagrams	Logistic model

(30 marks)

(b) State whether the following statements are True or False, and provide the reason if any of the statements are false.

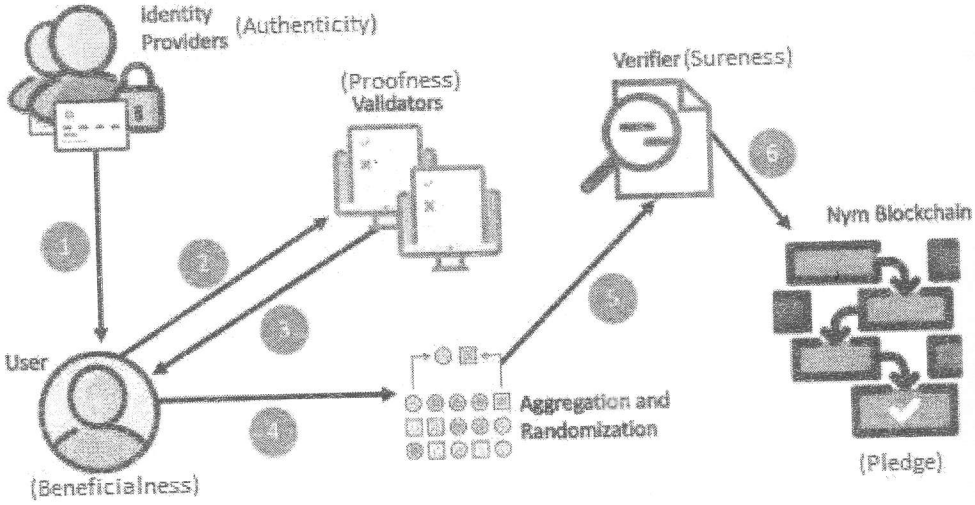
i. Mathematical modelling looping process helps to modify assumptions.

ii. Empirical models account quantitative changes for the same conditions.

iii. The time-diffusion model best predicts the stochastic behavior.

iv. Flow diagrams make the use of conservation laws. (20 marks)

(c) The following diagram shows a system of Nym network-level privacy for users in a networking structure. The nature of each component in the system is indicated within open and closed brackets. Each number in the diagram, from ① to ⑥, respectively represents the following: third-party certification, unsigned credential deposit, partially signed credential, credential validation, right-to-use, and checking double-spent and prevention.



i. Identify objects of the system and state the reason for your choice. (15 mark

ii. Specify each relationship between the objects individually. (12 mark

iii. Indicate whether the given system is open or closed. State the reason. (8 mark

iv. Identify an object to be an open system that can be viewed as a syst environment for another system. Explain your reasoning for this selecti (15 mark

Q3. (a) Assume that the population of cottony insects is targeted for removal using two methods: first, by introducing ladybird beetles as a biological control, and second, by applying pesticides.

Under the above measures, the interaction between the cottony insects and ladybird beetles is described by the following system of difference equations:

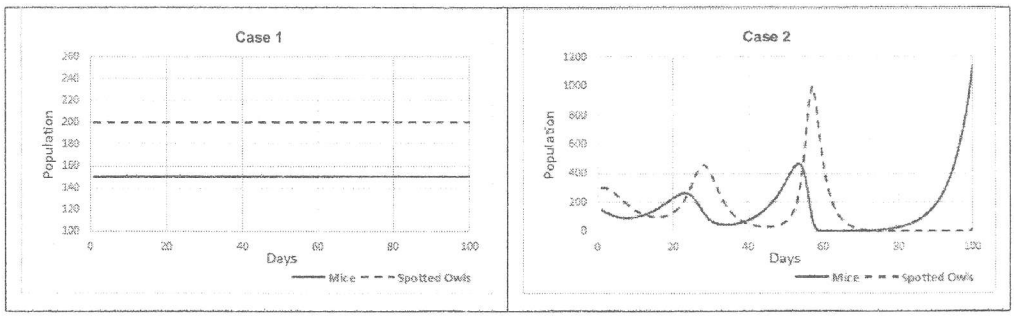
$$\begin{aligned} C_{n+1} &= C_n + \alpha C_n - \beta B_n C_n, \\ B_{n+1} &= B_n - \gamma B_n + \delta B_n C_n, \end{aligned}$$

where C_n and B_n represent the cottony insect and ladybird beetle population levels after n days, respectively.

i. Explain the last two terms in each model with respect to biological control and the application of pesticides. (25 marks)

ii. Find the equilibrium values for the above system. (35 marks)

(b) A predator-prey system demonstrates the long-term behavior of mice M and owls O for two different initial population sizes: (M,O)=(150,200) and (M, O)=(150,300), as illustrated in Case 1 and Case 2, respectively.



Is the system sensitive to initial conditions? Explain based on the long-term behavior of the populations of mice and owls. (40 marks)

Q2. (a) A solution to the logistic growth model can be expressed as

$$\ln P - \ln(M - P) = rMt + c,$$

where P is the population of a species at time t, M is the maximum population, and c and r are constants.

Suppose you are given a set of data representing the species population over continuous time.

i. Explain how would you propose an appropriate value for M based on the given data. (15 marks)

ii. Describe a method for finding the constants c and r. (25 marks)

(b) Given that the Malthusian model of population growth

$$P(t) = P_0 e^{k(t-t_0)},$$

where $P(t)$ denotes the population of a species at time t, P_0 indicates the initial population and k defines the rate of population. Suppose census data on the human population in New Zealand is given by

Year	Population
1994	3 611 526
2009	4 293 500
2024	5 338 500

i. Determine the value of k and explain what the value represents. (20 marks)

ii. How far your prediction of the population in 2024 is off the mark? (20 marks)

iii. State the reasons that you think caused the deviation in ii. (20 marks)

Q4. (a) A traffic density $\rho(x,t)$ can be modelled by

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}\left[v_m\left(1-\frac{\rho}{\rho_m}\right)\rho\right] = 0,$$

where x represents the position of a vehicle on a single-lane road at time t, v_m denotes the maximum velocity permitted on the road, and ρ_m indicates the maximum number of vehicles that can occupy a unit length of the road.

i. Using an appropriate transformation, nondimensionalise the above traffic model. (35 marks)

ii. Compare your nondimensionalised model with the provided model, and discuss the significance of nondimensionalisation. (15 marks)

(b) Given that a traffic flow model

$$\frac{\partial u}{\partial t} + \sqrt{u}\frac{\partial u}{\partial x} = 0, \quad x > 0,$$

with an initial condition $u(x,0) = x,\ x > 0$.

i. Solve the traffic flow model using the method of characteristics, and show that

$$u(x,t) = \frac{1}{2}(t^2 \pm t\sqrt{t^2+4x}) + x.$$

(40 marks)

ii. Draw the characteristics in the xt-plane? (10 marks)