EASTERN UNIVERSITY, SRI LANKA

FACULTY OF SCIENCE

First Year Second Semester Examination in Science - 2021/2022

(August/September 2024)

MT 1042 Vector Analysis

Answer all questions | Time : Two hours

---

1. (a) Find a unit vector parallel to the resultant of vectors $2\underline{i} + 4\underline{j} - 5\underline{k}$ and $\underline{i} + 2\underline{j} + 3\underline{k}$. [15 marks]

   (b) Find the area of a parallelogram having diagonals $3\underline{i} + \underline{j} - 2\underline{k}$ and $\underline{i} - 3\underline{j} + 4\underline{k}$. [30 marks]

   (c) If the midpoint of the consecutive sides of any quadrilateral are connected by straight lines then prove that the resulting quadrilateral is a parallelogram. [25 marks]

   (d) Find an equation for the plane passing through three point whose position vectors are given by $(2, -1, 1)$, $(3, 2, -1)$, $(-1, 3, 2)$. [30 marks]

2. (a) Find the curvature of the space curve $x = (3t - t^3), y = 3t^2, z = (3t + t^3)$.[45 marks]

   (b) What is the direction from the point $(1, 1, -1)$ when the directional derivative of $\phi = x^2 - 2y^2 + 4z^2$ is maximum and find the value of this maximum directional derivative. [25 marks]

   (c) Find the equation of the tangent plane to the surface $yz - zx + xy + 5 = 0$ at the point $(1, -1, 2)$. [30 marks]

3. (a) Show that $\underline{F} = (\sin y + z)\underline{i} + (x \cos y - z)\underline{j} + (x - y)\underline{k}$ is a conservative vector. Find the scalar potential function ϕ such that $\underline{F} = \underline{\nabla}\, \phi$. [45 mar

(b) If $\underline{a}$ is a constant vector. Prove that

$$\text{Curl}\frac{(\underline{a} \wedge \underline{r})}{r^3} = \frac{3(\underline{a} \cdot \underline{r})}{r^5}\, \underline{r} - \frac{\underline{a}}{r^3}.$$

[55 mar

4. (a) State the Green's theorem. [10 mar

Evaluate $\oint_C (y - \sin x)\, dx + \cos x\, dy$,

i. by directly;

ii. by using Green's theorem in the xy plane.

where C is the triangle joining points whose coordinates are $(0, 0)$, $\left(\frac{\pi}{2}, 0\right)$ ar $\left(\frac{\pi}{2}, 1\right)$ [60 mark

(b) If $F = (2x^2 - 3z)\underline{i} - 2xy\underline{j} - 4x\underline{k}$ then evaluate $\int\int\int_V \text{div}\underline{F}\, dV$, where V is t region bounded by $x = 0, y = 0, z = 0$ and $2x + 2y + z = 4$. [30 mark