EASTERN UNIVERSITY, SRI LANKA

THIRD EXAMINATION IN SCIENCE - 2018/2019

SECOND SEMESTER (Oct./ Nov., 2022)

AM 307 - CLASSICAL MECHANICS III

Answer all Questions **Time: Three hours**

---

1. Two frames of reference S and S' have a common origin O, and S' rotates with constant angular velocity $\underline{\omega}$ relative to S. At a time t, a particle P has position vector $\underline{r}$ with respect to O. If $\dot{\underline{r}}$ and $\ddot{\underline{r}}$ denote the velocity and acceleration of P relative to S' respectively, then prove that the acceleration of P relative to S is

$$\ddot{\underline{r}} + 2\underline{\omega} \wedge \dot{\underline{r}} + \underline{\omega} \wedge (\underline{\omega} \wedge \underline{r}).$$

An object is thrown downward with an initial speed v_0. Prove that after time t the object is deflected east of the vertical by the amount

$$\omega v_0 \sin \lambda t^2 + \frac{1}{3}\omega g \sin \lambda t^3,$$

where λ is the earth's co - latitude.

2. (a) Define the following terms:

   i. linear momentum;

   ii. angular momentum;

   iii. moment of force.

   (b) With the usual notations, obtain the equations of motion for a system of N particles in the following form:

   i. $M\underline{f}_G = \sum_{i=1}^{N} \underline{F}_i$;

   ii. $\dfrac{d\underline{H}}{dt} = \sum_{i=1}^{N} \underline{r}_i \wedge \underline{F}_i$, where $\sum_{i=1}^{N} \underline{h}_i = \underline{H}$ and $\underline{h}_i = \underline{r}_i \wedge m_i \underline{v}_i$;

   iii. $T = T_G + \frac{1}{2} M \underline{v}_G^2$.

(c) A uniform sphere of mass m and radius a is released from rest on a plane inclined at an angle θ to the horizontal. If the sphere rolls down without slipping, show that the acceleration of the center of the sphere is a constant and equal to $\frac{5}{7}g\sin\theta$.

3. (a) With the usual notation obtain the Euler's equations for the motion of the rigid body, having a point fixed, in the form:

$$A\dot{\omega}_1 - (B-C)\omega_2\omega_3 = N_1,$$

$$B\dot{\omega}_2 - (C-A)\omega_1\omega_3 = N_2,$$

$$C\dot{\omega}_3 - (A-B)\omega_1\omega_2 = N_3.$$

A body moves about a point O under no forces. The principle moment of inertia at O being $3A$, $5A$ and $6A$. Initially the angular velocity has components $\omega_1 = n$, $\omega_2 = 0$, $\omega_3 = 3$ about the corresponding principal axes. Show that at time t,

$$\omega_2 = \frac{3n}{\sqrt{5}}\tan\left(\frac{nt}{\sqrt{5}}\right).$$

4. Obtain the Lagrange's equations of motion using D'Alembert's principle for a conservative holonomic dynamical system.

A thin rod of mass m and length $2l$ stands on a frictionless floor and leans against a frictionless wall. Assume it remains in contact with both of them as it slides. Assume also that the point on the wall that is closest to the place where the rod touches the floor lies vertically below the point where the rod touches the wall. Find the Lagrange's equations of motion of the system.

5. With the usual notations, derive the Lagrang's equation for the impulsive motion from the Lagrange's equations for a holonomic system in the following form

$$\triangle\left(\frac{\partial T}{\partial \dot{q}_j}\right) = S_j \qquad j = 1, 2, ..., n.$$

A uniform rod AB of length $2a$ and mass m has a particle of mass M attached to the end B. It is at rest on a smooth horizontal table when an impulse I is applied at A in a direction perpendicular to AB, and in the plane of the table. Find the initial velocities of A and B and prove that the resulting kinetic energy is

$$\frac{2I^2(m+3M)}{m(m+4M)}.$$

6. (a) Define the poisson bracket.

Show that

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + [f,\ H],$$

for any function $f(q_i,\ p_i,\ t)$, where H is a Hamiltonian function.

(b) With the usual notations, prove that:

i. $\frac{\partial}{\partial t}[f,\ g] = \left[\frac{\partial f}{\partial t},\ g\right] + \left[f,\ \frac{\partial g}{\partial t}\right];$

ii. $[f,\ q_k] = -\frac{\partial f}{\partial p_k};$

iii. $[f,\ p_k] = \frac{\partial f}{\partial q_k}.$

(c) Show that, if f and g are constants of motion then their poisson bracket $[f,\ g]$ is a constant of motion.