EASTERN UNIVERSITY, SRI LANKA

FACULTY OF SCIENCE

Third Year Second Semester Examination in Science - 2019/2020

(October/November, 2023)

AM 307 - Classical Mechanics III

Answer all questions | Time allowed: Three hours

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1. Two frames of reference S and S' have a common origin O, and S' rotates with constant angular velocity $\underline{\omega}$ relative to S. At a time t a particle P has position vector $\underline{r}$ referred to O; and $\dot{\underline{r}}$ and $\ddot{\underline{r}}$ denote the velocity and acceleration of P relative to S' respectively.

   Prove that the acceleration of P relative to S is

   $$\ddot{\underline{r}} + 2\underline{\omega} \wedge \dot{\underline{r}} + \underline{\omega} \wedge (\underline{\omega} \wedge \underline{r}).$$

   An object is thrown downward with an initial speed v_0. Prove that after time t the object is deflected east of the vertical by the amount

   $$\omega v_0 \sin\lambda t^2 + \frac{1}{3}\omega g \sin\lambda t^3,$$

   where λ is the earth's co - latitude.

2. (a) With the usual notations, obtain the equations of motion for a system of N particles in the following forms:

   i. $M\underline{f}_G = \sum_{i=1}^{N} \underline{F}_i,$

   ii. $\frac{d\underline{H}}{dt} = \sum_{i=1}^{N} \underline{r}_i \wedge \underline{F}_i,$

   where $\sum_{i=1}^{N} \underline{h}_i = \underline{H}$ and $\underline{h}_i = \underline{r}_i \wedge m_i\underline{v}_i$.

   (State clearly the results that you may use)

   (b) A solid of mass M is in the form of a tetrahedron $OXYZ$, the edges OX, OY, OZ of which are mutually perpendicular, rests with XOY on a fixed smooth horizontal plane and YOZ against a smooth vertical wall. The normal to the

rough face XYZ is in the direction of a unit vector $\underline{n}$. A heavy uniform spher[e] of mass m and center C rolls down the face causing the tetrahedron to acquir[e] a velocity $-V\underline{j}$ where $\underline{j}$ is the unit vector along OY
If $\overrightarrow{OC} = \underline{r}$, then prove that

$$(M+m)V - m\dot{\underline{r}}\cdot\underline{j} = \text{constant}$$

and that

$$\frac{7}{5}\ddot{\underline{r}} = \underline{f} - \underline{n}(\underline{n}\cdot\underline{f}),$$

where $\underline{f} = \underline{g} + \dot{V}\underline{j}$ and $\underline{g}$ is the acceleration of gravity.

3. With the usual notation obtain the Euler's equations for the motion of the rig[id] body, having a point fixed, in the form:

$$A\dot{\omega}_1 - (B-C)\omega_2\omega_3 = N_1,$$

$$B\dot{\omega}_2 - (C-A)\omega_1\omega_3 = N_2,$$

$$C\dot{\omega}_3 - (A-B)\omega_1\omega_2 = N_3.$$

A body moves under no forces about a point O. The pricipal moment of inerti[a] at O being $6A$, $3A$, A. Initially the angular velocity of the body has component[s] $\omega_1 = n$, $\omega_2 = 0$, $\omega_3 = 3n$ about the principal axis. Show that at any latter tim[e] $\omega_2 = -\sqrt{5}n\tanh\sqrt{5}nt$.

4. Obtain the Lagrange's equations of motion using D'Alembert's principle for a conservative holonomic dynamical system.
A sphere of mass M and radius R rolls without slipping down the inclined plan[e] of a wedge shaped block of mass m that is free to move on a frictionless horizontal surface.

(a) Find the Lagrange's equations for this system subject to the force of gravity at the surface of the earth, given that all objects are initially at rest and the center of the sphere is at a distance H above the surface.

(b) Find the motion of the system by integrating Lagrange's equations.

5. With the usual notations, derive Lagrange's equation for the impulsive motion from Lagrange's equations for a holonomic system in the following form

$$\triangle\left(\frac{\partial T}{\partial \dot{q}_j}\right) = S_j \qquad j = 1, 2, ..., n.$$

A uniform rod AB of length $2a$ and mass m has a particle of mass M attached to the end B. It is at rest on a smooth horizontal table when an impulse I is applied at A in a direction perpendicular to AB, and in the plane of the table. Find the initial velocities of A and B and prove that the resulting kinetic energy is

$$\frac{2I^2(m+3M)}{m(m+4M)}.$$

6. (a) Define the poisson bracket.

Show that the Hamiltonian equations of the holonomic system may be written in the form

$$\dot{q}_k = [q_k, H], \qquad \dot{p}_k = [p_k, H],$$

and show that for any function $f(q_i,\ p_i,\ t)$, $\dfrac{df}{dt} = \dfrac{\partial f}{\partial t} + [f,\ H]$.

(b) Show that, if f and g are constants of motion then their poisson bracket $[f, g]$ is also a constant of motion.