EASTERN UNIVERSITY, SRI LANKA

THIRD EXAMINATION IN SCIENCE - 2018/2019

SECOND SEMESTER (July, 2022)

MT 3232 - General Topology

Answer all questions | Time : Two hours

1. (a) Define the term *topology* for a non-empty set X.

   (b) Let X be a non-empty set and $x \in X$. If $\mathcal{T}_x$ consists of X and all those subsets of X which do not contain x, show that $\mathcal{T}_x$ is a topology on X.

   (c) Define the term *closure* of a set A in a topological space $(X, \mathcal{T})$.

   (d) Let A and B be subsets of a topological space X. Prove the following:

   i. If $A \subseteq B$, then $\bar{A} \subseteq \bar{B}$;

   ii. $\overline{(A \cup B)} = \bar{A} \cup \bar{B}$;

   iii. $A \subset \overline{A}$;

   iv. $\overline{(\overline{A})} = \overline{A}$.

   [Hint: If $A \subseteq B$, then $A' \subseteq B'$.]

2. (a) Define the term *subspace* of a topological space.

   (b) Let A be a subset of a topological space (X, τ). Show that $\tau_A = \{A \cap G : G \in \tau\}$ is a subspace topology on A.

   (c) Define the term *continuous function* in topological spaces.

   (d) Let X and Y be topological spaces; and let $f : X \longrightarrow Y$ be a continuous function. Then prove that for each closed set C of Y, $f^{-1}(C)$ is closed in X.

   (e) Let X, Y and Z be topological spaces. If $f : X \longrightarrow Y$ and $g : Y \longrightarrow Z$ are continuous functions on the indicated spaces, then prove that the composite function $g \circ f : X \longrightarrow Z$ is a continuous function.

3. (a) Define the term *T_0-space.*

   (b) Prove that every subspace of a T_0-space is a T_0-space.

   (c) Let X be a T_0-space and Y is homeomorphic to X by homeomorphism $f : X \longrightarrow Y$
   Then prove that Y is a T_0-space.

   (d) Show that every discrete topological space is a T_0-space.

4. (a) Define the term *connected* in a topological space.

   (b) Prove that a subspace Y of a space X is disconnected if and only if there exists ope
   sets U and V in X such that

$$U \cap Y \neq \emptyset; \quad V \cap Y \neq \emptyset; \quad U \cap V \cap Y = \emptyset; \quad Y \subset U \cup V.$$