EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

THIRD YEAR EXAMINATION IN SCIENCE - 2021/2022

FIRST SEMESTER (Aug./Sep., 2024)

MT 3022 - FLUID DYNAMICS

Answer all questions — Time : Two hours

1. (a) Derive the continuity equation for a fluid flow in the form $\frac{D\rho}{Dt} + \rho\underline{\nabla}.\underline{q} = 0$, where ρ and $\underline{q}$ are the density and the velocity of the fluid respectively. Hence, show that the equation of continuity for an incompressible fluid in Cartesian coordinates reduces to the form $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$, where u, v and w are the cartesian components of the velocity.

   (b) Show that $\frac{k}{r^5}\left(3x^2 - r^2, 3xy, 3xz\right)$, where $r^2 = x^2 + y^2 + z^2$ and k is a constant, represents the velocity field in a possible fluid motion of an incompressible fluid and the the motion is irrotational.
   Also determine the streamlines.

2. (a) Let a gas occupy the region $r \leq R$, where R is a function of time t, and a liquid of constant density ρ lie outside the gas. If the velocity at $r = R$, the gas liquid boundary, is continuous, then show that the pressure p at a point $P(\underline{r}, t)$ in the liquid is given by

$$p = \Pi + \rho\left[\frac{1}{r}\frac{d}{dt}(R^2\dot{R}) - \frac{1}{2}\left(\frac{R^2\dot{R}}{r^2}\right)^2\right],$$

   where Π is the pressure at infinity. Here '.' denotes differentiation with respect to t.

(b) A spherical gas bubble whose center is fixed and whose radius at time t is $a + b\cos\omega t$, where a, b and ω are constants, is surrounded by an infinite mass of liquid of uniform density ρ, on which no body forces act. Prove that the pressure at the surface of the sphere is $\Pi + \frac{1}{4}b\rho\omega^2\left[b - 5b\cos 2\omega t - 4a\cos\omega t\right]$. Find the least value of this pressure.

3. (a) Let a three-dimensional doublet of strength μ be situated at the origin. Show that the velocity potential Φ at a point $P(r, \theta, \phi)$, in spherical polar coordinates, due to the doublet is given by $\Phi = \mu r^{-2}\cos\theta$.

(b) Three-dimensional doublets of strengths μ_1, μ_2 are situated at A and B whose cartesian coordinates are $(0, 0, a)$ and $(0, 0, b)$, their axes being directed towards and away from the origin, respectively. Show that the condition for no transport of fluid across the surface of sphere $x^2 + y^2 + z^2 = ab$ is $\frac{\mu_1}{\mu_2} = \left(\frac{a}{b}\right)^{3/2}$

4. Write down the Bernoulli's equation for steady motion of an incompressible inviscid fluid.

Given that an incompressible inviscid fluid of constant density ρ fills the region of space on the positive side of the x axis, which is a rigid boundary and there be a source of strength m at the point $(0, a)$ and an equal sink at $(0, \text{b})$, where $a > b > 0$. If the pressure on the negative side is the same as the pressure at infinity, show that the resultant pressure on the boundary is $\frac{\pi\rho m^2(a-b)^2}{2ab(a+b)}$.