EASTERN UNIVERSITY, SRI LANKA

SECOND YEAR FIRST SEMESTER EXAMINATION IN SCIENCE – 2021/2022

(March/April – 2024)

PH 2012 MECHANICS

Time: 02 hours

Answer ALL Questions

You may find the following useful.

Radius of the Earth is 6.4×10^{6} m

Mass of the Earth is 6.0×10^{24} kg

Universal Gravitational Constant is 6.67×10^{-11} Nm^2kg^{-2}

1.

(a) Considering that the exhaust gases are ejected at a constant velocity relative to the rocket, show that the equation of motion of a rocket is given by;

$$F_{ext} = m\frac{dv}{dt} + v\frac{dm}{dt}$$

where the symbols have their usual meanings. (40 Marks)

(b) A spacecraft is moving in a straight line in deep space in a straight line with speed $2u$. At time $t = 0$ the mass of the space craft is M and at that instant, the engine of the space craft is fired in a direction opposite to that of the motion of the space craft. Fuel is ejected at a constant mass rate of k with speed u relative to the space craft. At time t, the mass of the space craft become as m and its speed as v.

(i) Use impulse-momentum principle to show that;

$$\frac{dv}{dt} = \frac{uk}{M - kt}$$

(30 Marks)

(ii) Hence show that the speed of the spacecraft in terms of u when the mass of the space craft becomes $1/3$ of its initial mass is;

$$v = (2 + \ln 3)u$$

, (30 Marks)

2.

(a) Define the term gravitational field strength. (10 Marks)

(b) The Gravitational force exerted by the Earth maintains a satellite in a circular orbit of radius r. Find the magnitude of gravitational force and centripetal force between the Earth and the satellite. Hence show that the radius of the orbit r in terms of the orbiting period T of the satellite is given by;

$$r^3 = \frac{GM}{4\pi^2}T^2$$

Where the symbols have their usual meanings. (20 Marks)

If the orbital period of a geostationary satellite is equal to the period of rotation of the Earth about its axis then find;

(i) The height of the satellite above the Earth's surface and,

(ii) The speed of the satellite in its orbit.

(30 Marks)

(c) Another satellite named 'Kalpana' is in an orbit of radius 6.7×10^6 m around the earth. This satellite is to be increased to escape velocity.

(i) Explain the term "escape velocity"

(ii) Derive and show that an expression for the escape velocity is given by;

$$v = \sqrt{\frac{2GM}{r}},$$

where the symbols have their usual meanings.

(iii) Calculate the escape velocity of the satellite 'Kalpana'.

(40 Marks)

3.

(a) A particle of mass m in a central force field $\overrightarrow{F(r)}$ moves with a constant angular momentum $\bar{L}$ about the force center. Show that the general equation of the particles' orbit is given by;

$$\frac{d^2u}{d\theta^2} + u = -\frac{m}{L^2u^2}F\left(\frac{1}{u}\right)$$

Where $r$and θ are the plane polar co-ordinates of the particle, $u = \frac{1}{r}$, and the equation of motion can be written as, $F(r) = m(\ddot{r} - r\dot{\theta}^2)$.

(45 marks)

(b) An object of unit mass orbits in a central potential $V(r)$. Its orbital radius is $r = ae^{-b\theta}$, where θ the azimuthal angle measured in the orbital plane with a and b are constants.

(i)Find an expression for the central potential $V(r)$ and,

(45 marks)

(ii) hence show that for a unit mass, the central potential is given by,

$$V(r) = -L^2\left(\frac{1+b^2}{2r^2}\right)$$

(10 marks)

4.

(a) State and prove the Gauss's Theorem for an internal point enclosed by a closed surface. (40 marks)

(b) Calculate the gravitational field strength and gravitational potential at a point P due to a uniform solid sphere of radius R and mass M where P is placed at a distance r from the center of the solid sphere, for the following cases.

(i) When P is outside the solid sphere, and (20 marks)

(ii) When P is inside the sphere. (35 marks)

(iii) Sketch the variation of gravitational field strength with the distance from the center of the sphere. (05 marks)

...End of Exam...