EASTERN UNIVERSITY, SRI LANKA

FACULTY OF SCIENCE

Second Year First Semester Examination in Science - 2021/2022

(Mar./Apr., 2024)

MT 2212 - Metric Spaces

Answer all questions **Time : Two hours**

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1. Define the term ***metric space***. [10 marks]

   (a) Let X be any non-empty set and d is a metric defined over X. Show that $\left(\frac{d}{1+d}\right)$ is also a metric over X. [25 marks]

   (b) Let (X, d) be a metric space and let A be a subset of X. Define the terms **open ball, interior point of** A**, and interior** (A°) **of** A. [15 marks]

   In a metric space, prove the following:

   i. the intersection of a finite number of open sets is open; [30 marks]

   ii. the union of a finite number of closed sets is closed. [20 marks]

2. Define the terms ***separated*** and ***disconnected*** as applied to subsets of a metric space. [10 marks]

   (a) Prove that two closed subsets of a metric space are separated if and only if they are disjoint. [20 marks]

   (b) A metric space (X, d) is disconnected if and only if it can be written as a union of two non-empty disjoint closed sets. [35 marks]

   (c) Continuous image of the connected set is connected. [35 marks]

3. Define the term ***compact*** as applied to subsets of a metric space. [15 marks]

(a) Show that every compact subset of a metric space is closed and bounded. [45 marks]

(b) Prove that every closed subset of a compact space is compact. [20 marks]

(c) Let f be a continuous function on a metric space (X, d). Prove that if $A \subseteq X$ is compact, then $f(A)$ is compact. [20 marks]

4. Define what is meant by a ***continuous function*** between two metric spaces. [10 marks]

(a) Let (X, d_x) and (Y, d_y) be two metric spaces, and let $f : X \to Y$ be a function. Prove that f is continuous at $a \in X$ if and only if for every sequence $\{a_n\}$ in X converging to a implies that $\{f(a_n)\}$ converges to $f(a)$. [30 marks]

(b) Let (X, d_x) and (Y, d_y) be two metric spaces and let $f : X \to Y$ be a function. Show that the following statements are equivalent:

i. f is continuous;

ii. $f^{-1}(G)$ is open in X whenever G is open in Y;

iii. $f^{-1}(F)$ is closed in X whenever F is closed in Y; [60 marks]