EASTERN UNIVERSITY, SRI LANKA

SECOND YEAR SECOND SEMESTER EXAMINATION IN SCIENCE – 2020/2021

(Feb/Mar – 2024)

PH 2062 ELECTROMAGNETIC PHENOMENA

Time: 02 hours

Answer ALL Questions

You may find the following information useful.

- The vector identities

$$\vec{\nabla}.(\vec{A} \times \vec{B}) = \vec{B}.(\vec{\nabla} \times \vec{A}) - \vec{A}.(\vec{\nabla} \times \vec{B})$$

$$\vec{\nabla}.(f\vec{A}) = f.(\vec{\nabla}.\vec{A}) + \vec{A}.(\vec{\nabla} f)$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}.(\vec{\nabla}.\vec{A}) - \nabla^2 \vec{A}$$

$$\vec{\nabla} \times \vec{\nabla} f = 0, \text{ where } f \text{ is a scalar function.}$$

- Gauss divergence theorem

$$\oint_S \vec{A}.\overrightarrow{da} = \int_V \vec{\nabla}.\vec{A} d\tau$$

- Strokes theorem

$$\oint_C \vec{A}.\overrightarrow{dl} = \int_S (\vec{\nabla} \times \vec{A})\overrightarrow{da}$$

The symbols have their usual meaning.

1. (a) Derive the first Maxwell's equation of the electromagnetic field in

free space

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$$

on the basis of Gauss's theorem in electrostatics. (20 marks)

(b) Derive the second Maxwell's equation of the electromagnetic field

$$\vec{\nabla}.\vec{B} = 0$$

on the basis of Biot-Savart law for magnetic field. (30 marks

(c) State the Faraday's law of electromagnetic induction, and henc

derive the third Maxwell's equation of the electromagnetic field

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

(20 mark

(d) State the Ampere's circuital law, and hence derive the complete

form of the fourth Maxwell's equation of the electromagnetic field i

free space

$$\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

where the term $\varepsilon_0 \frac{\partial \vec{E}}{\partial t}$ refers to the displacement current density.

(30 mark

2. Starting from the Maxwell's equations obtain the wave equation for the electric field and show that the velocity c of an electromagnetic wave in free space is given by;

$$c^2 \varepsilon_0 \mu_0 = 1$$

where ε_0 and μ_0 are the electric permittivity and magnetic permeability of free space respectively. (40 marks)

The electric and magnetic components of a plane electromagnetic wave travelling in vacuum is described by;

$$\vec{E} = E_0 e^{i(\omega t - kz)} \hat{\imath}$$

$$\vec{B} = B_0 e^{i(\omega t - kz)} \hat{\jmath}$$

where k is the wave vector, ω is the angular frequency of the wave, and $\hat{\imath}$ and $\hat{\jmath}$ are the unit vectors along x- and y- axis respectively.

Using the appropriate Maxwell's equation show that

$$\frac{E_0}{B_0} = \frac{\omega}{k}.$$

(60 marks)

3. The electric field $\vec{E}$ in a matter satisfies the following wave equation

$$\nabla^2 \vec{E} = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2}.$$

Consider the solution of the above equation as $\vec{E} = \vec{E}_o e^{i(\omega t - kx)}$, where the symbols have their usual meaning.

Show that k and ω satisfy the dispersion relation:

$$\omega^2 \mu \varepsilon = i\omega\mu\sigma + k^2.$$ (25 marks)

Consider the electric component of the wave travelling in an ionized gas with $\varepsilon = \varepsilon_o$ and $\mu = \mu_o$.

i. Show that the dispersion relation becomes as

$$\frac{k}{\omega} = \sqrt{\left(1 - \frac{\omega_g^2}{\omega^2}\right)\varepsilon_o \mu_o}$$

where, ω_g is the frequency of ionized gas.

ii. Find the frequency of the ionized gas.

iii. Determine the refractive index of the medium, when the electron concentration is $2.5 \times 10^{10}\ m^{-3}$ and the frequency of wave is 3 MHz.

Here, the symbols have their usual meaning.

(75 marks)

4.

(a) State the domain theory of ferromagnetic materials, which response to an applied magnetic field.

Sketch a hysteresis loop traced out by magnetization of a ferromagnetic material as the applied magnetic field is cycled. Mark the following characteristic points: (i) Saturation magnetization, (ii) Residual magnetization, (iii) Coercivity in the hysteresis loop and describe the physical significance of each point when the applied magnetic field is cycled.

Differentiate soft magnetic materials from hard magnetic materials with regard to their hysteresis effect, and state at least three applications for each type of magnetic materials.

(60 marks)

(b) In superconductors, electrons continue to accelerate in the presence of applied electric field E. Show that the equation of motion of electrons in the superconducting state is given by

$$\frac{dJ_s}{dt} = \left(\frac{n_s e^2}{m}\right) E,$$

where J_s is the current density of superconducting electrons, n_s is the number density of superconducting electrons, e is the change of an electron, and m is the mass of an electron.

Hence, show that the London penetration depth λ_L of an electron in a superconducting material is given by

$$\lambda_L = \sqrt{\frac{m}{\mu_0 n_s e^2}}$$

where μ_0 is the permeability of free space.

(40 marks)

-End of Exam-