EASTERN UNIVERSITY, SRI LANKA
FACULTY OF SCIENCE
Second Year Second Semester Examination in Science-2020/2021
(Feb./Mar., 2024)
MT 2042 - Classical Mechanics I

Answer all questions **Time : Two hours**

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1. State the radial and transverse components of the velocity and acceleration of a particle in terms of the polar coordinates (r, θ). **[10 marks]**

   (a) A particle moves in a plane such that the radiant and transverse component of velocity are λr and $\mu\theta$ respectively. Show that the radiant and transverse component of acceleration are $\lambda^2 r - \left(\frac{\mu^2\theta^2}{r}\right)$ and $\mu\theta\left(\lambda + \frac{\mu}{r}\right)$ respectively. **[20 marks]**

   (b) A light inextensible string AB of length $2a$ passes through a smooth ring at a point O, on a smooth horizontal table and two particles, each of mass m, attached to it's ends A and B. Initially the particles lie on the table with $OA = OB = a$ and AOB is a straight line. A velocity u is given to the particle A in a direction perpendicular to OA. Prove that if r and θ are the polar co-ordinates of A at a time t with respect to the origin, then

   i. $2\dfrac{\mathrm{d}^2 r}{\mathrm{d}t^2} - \dfrac{a^2u^2}{r^3} = 0,$

   ii. $2r\dfrac{\mathrm{d}r}{\mathrm{d}t} = u\sqrt{2(r^2 - a^2)},$

   iii. $r^2 = a^2 + \dfrac{1}{2}u^2t^2.$

   Find the velocity of A at the instant when B reaches at a point O. **[70 marks]**

2. Define the following terms:

   (a) Linear Momentum;

   (b) Angular Momentum. **[20 marks]**

A particle is projected horizontally along the inner surface of a smooth cone, whos axis is vertical and vertex upwards. Find the pressure at any point in terms of th depth below the vertex. Show that the particle will leave the cone at the depth belo the vertex given by $\left(\frac{V^2h^2}{g\tan^2\alpha}\right)^{\frac{1}{3}}$, where h is the initial depth from the vertex and is an initial velocity, and α is the semi angle of the cone. [80 mark

3. State the differential equation for the equilibrium of the string at any point. Usi that, derive the differential equation in the directions $\underline{t}$, $\underline{n}$ and $\underline{b}$. [20 mark

The rough rigid wire is in the form of catenary with parameter c. It is fixed i a vertical plane, with its directrix is horizontal and its vertex is above. A unifor heavy chain of length c is in limiting with one end at the vertex of the wire. Prove th co-efficient between the wire and chain is $\frac{\log 4}{\pi}$. [80 mark

4. State the following with usual notation:

   (a) Macaulay's Notation;

   (b) Clapeyron's Equation. [20 mark

A heavy uniform elastic rod rests on 5 supports which are in a horizontal line. Tw of the supports are at the ends of the rod. One is at a middle point and bisects th distance between the middle point and the ends. Show that the bending moment the center and at each of the support next to its are

$$\frac{Wl}{56} \text{ and } \frac{3Wl}{112},$$

where W is the weight of the rod and $4l$ is the length of the rod. Show that t reaction on the points of supports are in the ratio $11 : 26 : 32$. [80 mark