EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

FIRST EXAMINATION IN SCIENCE - 2011/2012

FIRST SEMESTER (Jan./Feb., 2014)

AM 106 - TENSOR CALCULUS

(Proper & Repeat)

Answer all questions Time : One hour

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1. (a) Define the following terms:

   i. covariant tensor,

   ii. contravariant tensor.

   (b) Write the transformation equation for the following tensors:

   i. A_k^{pt},

   ii. B_{tk}^{pqr},

   iii. D_{ptk}^{m}.

   (c) Show that the contraction of the outer product of the tensors A^p and B_q is an invariant.

   (d) The covariant components of a tensor of rank one in rectangular coordinate system are $2x - z,\ x^2y,\ yz$. Find its covariant components in spherical coordinate (r, θ, ϕ).

2. (a) i. Define the Christoffel's symbols of the first and second kind.

ii. Determine the Christoffel's symbols of the second kind for the metric

$$ds^2 = a^2 d\theta^2 + a^2 \sin^2\theta d\phi^2$$

where a is a constant, and find the corresponding differential equation for geodesic.

(b) i. Write down the covariant derivative of the tensor A^i_{jk}.

ii. With the usual notation, prove that

$$\frac{\partial g_{pq}}{\partial x^m} = [pm,\ q] + [qm,\ p].$$

Hence deduce that the covariant derivative of a metric tensor g_{jk} is zero.

iii. Using the covariant derivative of a metric tensor, prove that

$$\Gamma^e_{\ ca} = \frac{1}{2} g^{eb}[\partial_c(g_{ab}) + \partial_a(g_{cb}) - \partial_b(g_{ca})], \quad \text{where } \partial_i = \frac{\partial}{\partial x^i}.$$