EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

FIRST EXAMINATION IN SCIENCE(2016/2017)

FIRST SEMESTER (Aug./Sept., 2018)

AM 106 - TENSOR CALCULUS

Answer all question **Time: One hour**

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1. (a) Define what is meant by symmetric and skew symmetric tensor A_{pq}.

   If $ds^2 = g_{jk}\,dx^j\,dx^k$ is an invariant, then prove that g_{jk} is a symmetric covariant tensor of rank two.

   (b) Let A^{rst}_{pq} be a tensor. If $p = t$, $q = s$ then show that A^{rqp}_{pq} is a tensor. What is its rank?

   (c) The covariant components of a tensor in rectangular co-ordinate system are $x^2 - y$, $2x - z^2$, xz. Find its covariant components in clyndrical co-ordinate system.

2. (a) Define the *Christoffel's symbols of the first and second kind.*

   (b) With the usual notations, prove the following:

   i. $[p\,q, r] = g_{rs}\Gamma^s_{p\,q}$;

   ii. $[p\,m, q] + [q\,m, p] = \dfrac{\partial g_{p\,q}}{\partial x^m}$;

   iii. $\dfrac{\partial g^{p\,q}}{\partial x^m} + g^{p\,n}\Gamma^q_{m\,n} + g^{q\,n}\Gamma^p_{m\,n} = 0.$

   (c) Show that the non-vanishing Christofel's symbols of the second kind in cylindrical coordinate (ρ, ϕ, z) are given by

$$\Gamma^1_{22} = -\rho, \quad \Gamma^2_{21} = \frac{1}{\rho}, \quad \Gamma^2_{12} = \frac{1}{\rho},$$

   where $x^1 = \rho$, $x^2 = \phi$, $x^3 = z$.