EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

FIRST EXAMINATION IN SCIENCE(2015/2016)

FIRST SEMESTER (Jul./Aug., 2017)

AM 106 - TENSOR CALCULUS

Answer all question **Time: One hour**

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1. (a) Define what is meant by the terms *symmetric tensor*, *skew symmetric tensor* and *invariant.*

   If $ds^2 = g_{jk}\,dx^j\,dx^k$ is an invariant, then prove that g_{jk} is a symmetric covariant tensor of rank two.

   (b) The covariant components of a tensor in rectangular coordinate system are $2x - y$, x^2y and yz. Find its covariant components in cylindrical coordinate system.

   (c) Let A^{rst}_{pq} be a tensor. Choose $p = t$, $q = s$ and show that A^{rqp}_{pq} is a tensor. What is its rank?

2. (a) Define the *Christoffel's symbols of the first and second kind.*

   (b) With the usual notations, prove the following:

   i. $[p\,q, r] = g_{rs}\Gamma^s_{p\,q}$;

   ii. $\dfrac{\partial g_{rs}}{\partial r^m} = [rm, s] + [sm, r]$;

   iii. $\dfrac{\partial g^{r\,s}}{\partial x^m} + g^{r\,n}\Gamma^s_{m\,n} + g^{s\,n}\Gamma^r_{m\,n} = 0.$

   (c) Show that the non-vanishing Christoffel's symbols of the second kind in spherical coordinate (r, θ, ϕ) are given by

   $\Gamma^1_{22} = -r$, $\Gamma^2_{21} = \Gamma^2_{12} = \Gamma^3_{13} = \Gamma^3_{31} = \dfrac{1}{r}$, $\Gamma^1_{33} = -r\sin^2\theta$, $\Gamma^2_{33} = -\sin\theta\cos\theta$ and $\Gamma^3_{23} = \cot\theta$.