EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

FIRST EXAMINATION IN SCIENCE - 2011/2012

FIRST SEMESTER (Jan./Feb., 2014)

AM 151 - MATHEMATICA

(Proper & Repeat)

Answer all questions **Time : Two hours**

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1. (a) If x is an approximation to $\sqrt{a}$, it can be shown that $\frac{1}{2}\left(x+\frac{a}{x}\right)$ is a better approximation. Use **NestList** to observe the first 10 approximations obtained in computing $\sqrt{3}$, starting with $x = 100$.

   (b) The 20^{th} prime number is 71. Find all the numbers less than 71 which are not prime.

   (c) i. Factor $4x^{\frac{2}{3}} + 8x^{\frac{1}{3}} + 4$.

   ii. Simplify the given expression $\frac{(\frac{2}{x}-3)}{1-\frac{1}{x-1}}$.

   (d) i. Evaluate $\int \frac{x^5+x^2+x+2}{(x^2+1)^2}dx$.

   ii. Evaluate $\lim_{x\to 1^+}\left(\frac{1}{\ln x}-\frac{1}{x-1}\right)$.

   iii. Find the third derivative of the function $g(t) = t^3 - \sqrt{t} + e^{-2t}$.

(e) Let $A = \begin{pmatrix} 3 & -1 & 2 & 1 \\ 2 & 7 & -3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -1 \\ 3 & 2 \\ -4 & -3 \\ 0 & -2 \end{pmatrix}$. Find the A^T, B^T and verify that $(AB)^T = B^T\, A^T$.

(f) Find all solution of the equation $x^3 = 2x + 1$.

(g) Consider the parallelepiped with sides $a = j+k$, $b = 2i+j+3k$ and $c = i+j+2k$.

i. Find the volume.

ii. Find the area of the face determined by b and c.

2. (a) Plot the graph of the function $f(x) = \begin{cases} 1 - x^2, & \text{if } \quad x < 1 \\ \dfrac{1}{x}, & \text{if } \quad x \geq 1 \end{cases}$, and indicate where the function is discontinuous .

(b) Find the equation of the tangent line to the curve $y = \dfrac{1-x}{1+x}$ at the point $(2, -1/3)$ and the sketch the graph of the tangent line.

(c) Plot the graph showing the region under the curve $y = x^4$ from $x = -1$ to $x = 2$, and then find the area of the region.

(d) Plot the polar curve represented by $r = 2$ when

i. $0 \leq \theta \leq \pi$,

ii. $-\pi/4 \leq \theta \leq \pi/4$,

iii. $-2\pi \leq \theta \leq 2\pi$,

where $r = \sqrt{x^2 + y^2}$.

(e) Find all the critical numbers for the function $f(x) = x^{4/5}(x-4)^2$.

(f) Consider the sequence $\left\{(-1)^{n-1} \dfrac{n+2}{5^n}\right\}_{n=1}^{\infty}$.

i. List the first 7 terms of the sequence .

ii. Determine whether the sequence converges.

iii. Find the sum of the first 7 terms of the sequence.

iv. Find the sum of the first n terms of the sequence.

v. Find the sum of the entire sequence (from 1 to ∞).

3. (a) Find the area of the surface generated by rotating the curve $y = e^x$, $0 \leq x \leq 1$, about the y axis.

   (b) Suppose a curve C is defined by the parametric equation $x = t^2$, $y = t^3 - 3t$.

      i. Plot the curve.

      ii. Find the equation(s) of the tangent line(s) to the curve at the point $(3, 0)$.

      iii. Plot the tangent line(s) at the point $(3, 0)$.

   (c) Use mathematica to find the general solution of the logistic equation

$$\frac{dy(t)}{dt} = (r - ay(t))y(t).$$

      i. Approximate the population using $r = 0.03$, $a = 0.0001$, and $y(0) = 5.3$.

      ii. Investigate the behavior of the solution when the initial population is varied.