EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

FIRST EXAMINATION IN SCIENCE - 2008/2009

FIRST SEMESTER (Mar./May, 2010)

MT 101 - FOUNDATION OF MATHEMATICS

Answer all questions — Time : Three hours

1. (a) Prove the following equivalences using the laws of algebra of proposition:

   i. $(\sim p \wedge q) \vee (p \vee \sim q) \equiv t$,

   ii. $[p \vee (q \wedge r)] \vee \sim [(\sim q \wedge \sim r) \vee r] \equiv p \vee q$,

   where t denotes a proposition which is always true.

   (b) Using valid argument forms, draw a valid conclusion to the premises given below. Give reasons at each step.

$$\sim p \rightarrow q \wedge \sim r$$
$$s \rightarrow r$$
$$u \rightarrow \sim p$$
$$\sim w$$
$$u \vee w$$

2. (a) For any sets A and B, prove that $A \triangle B = (A \cup B) \setminus (A \cap B)$.

   Hence show that:

   i. $A \triangle B$ and $A \cap B$ are disjoint,

   ii. $A \cup B = (A \triangle B) \cup (A \cap B)$.

   (b) For any sets A, B and C, prove that:

   i. $A \times (B \cap C) = (A \times B) \cap (A \times C)$,

   ii. $A \times (B \setminus C) = (A \times B) \setminus (A \times C)$.

3. (a) Let R be a relation defined on $\mathbb{C} \setminus \{0\}$ by $z_1 R z_2 \Leftrightarrow z_1\bar{z}_1(z_2 + \bar{z}_2) = z_2\bar{z}_2(z_1 + \bar{z}_1)$, where $\mathbb{C}$ denotes the set of all complex numbers. Prove that R is an equivalence relation.

If a is a non-zero real number, show that R−class of a is a circle with centre at $(\frac{1}{2}a, 0)$ and radius $\frac{1}{2}a$.

(b) Let R be an equivalent relation on a set A. Prove the following:

i. $[a] \neq \Phi \ \forall a \in A$,

ii. $aRb \Leftrightarrow [a] = [b]$,

iii. either $[a] = [b]$ or $[a] \cap [b] = \Phi \ \ \forall a \in A$.

4. (a) Define the following terms:

i. *injective mapping*,

ii. *surjective mapping*,

iii. *inverse mapping*.

(b) The functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ are defined by

$$f(x) = \begin{cases} 1 - x, & \text{if } x \geq 0; \\ x^2, & \text{if } x < 0; \end{cases} \quad \text{and } g(x) = \begin{cases} x, & \text{if } x \geq 0; \\ x - 1, & \text{if } x < 0. \end{cases}$$

Find the formula for $f \circ g$.

Show that $f \circ g$ is a bijection and give a formula for $(f \circ g)^{-1}$

5. (a) Let $f : X \to Y$ be a mapping. Prove that f is injective if and only if $f(A) \cap f(X \setminus A) = \Phi \ \ \forall A \subseteq X$.

(b) i. Prove that every partially ordered set has at most one first element.

ii. Show that first element of every partially ordered set is a minimal element.

Is the converse true? Justify your answer.

6. (a) State *division algorithm*.

For $n \geq 1$, prove that $n(n + 1)(2n + 1)/6$ is an integer.

(b) i. Using the Euclidean algorithm find integers x and y satisfying

$$gcd(119, 272) = 119x + 272y.$$

ii. Determine all positive integer solutions of the Diophantine equation

$$18x + 5y = 48.$$