



EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

SECOND EXAMINATION IN SCIENCE - 2008/2009

FIRST SEMESTER (Mar./May, 2010)

MT106 - TENSOR ANALYSIS

(PROPER)

Answer all questions

Time : One hour

1. (a) The components of a contravariant tensor in the x coordinate system are

$$A^{11} = 4, A^{12} = A^{21} = 0 \text{ and } A^{22} = 7.$$

Find its components in the \bar{x} coordinate system, where

$$\bar{x}^1 = 4(x^1)^2 - 7(x^2)^2, \quad \bar{x}^2 = 4x^1 - 5x^2.$$

- (b) Let $ds^2 = g_{jk} dx^j dx^k$ is an invariant. Show that g_{jk} is a symmetric covariant tensor of rank two.
- (c) Prove that $A_{pq} x^p x^q = 0$ if A_{pq} is a skew-symmetric tensor.
- (d) Express the relationship between the following associated tensors:
- A^{ijk} and A_{pqr}
 - $A_{j \cdot l}^k$ and A^{qkr}
 - $A_{\cdot \cdot n}^{qm \cdot tk}$ and $A_{\cdot r \cdot p}^{q \cdot st}$

2. (a) Define the Christoffel's symbols of the first and second kind.

(b) Determine the Christoffel's symbols of the second kind for the metric

$$ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2.$$

(c) With the usual notations, prove the following:

i. $\frac{\partial g_{pq}}{\partial x^m} = [pm, q] + [qm, p],$

ii. $\frac{\partial g^{pq}}{\partial x^m} = -g^{pn} \Gamma_{mn}^q - g^{qn} \Gamma_{mn}^p.$

(d) Prove that the covariant derivatives of g_{jk} , g^{jk} and δ_k^j are zero.