EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS
FIRST EXAMINATION IN SCIENCE - 2008/2009
FIRST SEMESTER(December, 2009)
MT 151 - MATHEMATICA
(PROPER & REPEAT)

Answer all Questions **Time: Two hours**

1. (a) Find the greatest common divisor of 24, 40 and 48. [10 marks]

   (b) Find the smallest integer greater than or equal to -11.5. [10 marks]

   (c) Compute the numerical value of $\pi^3 + \pi + 7e$ correct to 8 digits. [15 marks]

   (d) Write down a command to return a uniformly distributed random complex number in the rectangle determine by the vertices $1+i$ and $3+4i$.

   [15 marks]

   (e) Compute the numerical approximation of

   $$\left(1+\frac{1}{2}\right)\left(1+\frac{1}{2}+\frac{1}{3}\right)\cdots\left(1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{10}\right)$$

   [25 marks]

   (f) Join the string **MT** followed by **151**. Moreover, display the second character from the last of the jointed string. [15 marks]

   (g) Without using assignment operators find the value of the function $3x^2 + y^3$ at $x = 2$ and $y = 1$. [10 marks]

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2. (a) Find

i. $\lim_{x\to\infty}\left(1+\frac{a}{x}\right)^x$,

ii. a numerical value for the integral $\int_0^1 \cos^2 x^2\, dx$,

iii. the second derivative of $\cos ax$ at $x = 0$,

where a is a constant. [25 marks]

(b) Plot the graph of the function, $f(x) = \sin x \cos 2x, -\pi \leq x \leq \pi$, with the following options:

i. 0.02% solid part and 0.05% space of the graph,

ii. width of line 0.003% of the standard graph, and

iii. the title, **The graph of sin$x$ cos2x.** [25 marks]

(c) Plot the functions:

$$f(x, y) = x^2 + y^2, \quad g(x, y) = 16 - (x^2 + y^2), \quad -3 \leq x, y \leq 3,$$

in the same set of axis using a suitable command. [25 marks]

(d) Using **Table** command create a list of 50 random integers in the interval [0,10] in 3-dimension. Furthermore, plot the above list of points as a pattern of dots of size 0.02. [25 marks]

3. (a) Using the statement **Which** define the function

$$f(x) = \begin{cases} -1, & x \leq -1; \\ -[1-(x+1)^2]^2, & -1 < x \leq 0; \\ -[1-(x-1)^2]^2, & 0 < x \leq 1; \\ 1, & x > 1, \end{cases}$$

and evaluate the functional value at $x = \ln 2$, numerically. [20 marks]

(b) Solve the system of differential equations:

$$\frac{dx}{dt} = \cos(xy)$$
$$\frac{dy}{dt} = x$$

for $0 \leq t \leq 5$ with initial conditions $x(0) = 1$ and $y(0) = -1$. Find a numerical solution at $t = 3$ by defining two separate functions for $x(t)$ and $y(t)$.

[30 marks]

(c) Assume that the relation

$$C_k = \frac{2k+1}{2}\int_{-1}^{1} x^n P_k(x)dx, \quad k = 0, 1, \ldots, n;$$

can be used to find the coefficients C_k for x^n using k^{th}-order Lengendre polynomial.

A mathematica function **poly** given below is **not** a meaningful option for this purpose. Find the errors within the coding and correct it to find the coefficients of x^{10}.

$$\begin{aligned}\text{poly}[n] := {} & \text{Module}[\{c, k\}, c = .\text{table}[0, \{m, 0, n\}];\\ & \text{for}[k = 0, k = k + 1, k <= n,\\ & c[[k+1]] = \frac{2k+1}{2}\ \text{Integrate}[x^n \text{LengendreP}_k(x), \{x, -1, 1\}],\\ & \text{Returns}[k]]\end{aligned}$$

[20 marks]

(d) Using **Module** command write a programme to find the sum of a given list of numbers and check your coding by defining an arbitrary list of numbers.

[30 marks]