



EASTERN UNIVERSITY, SRI LANKA

FIRST EXAMINATION IN SCIENCE - 2008/2009

FIRST SEMESTER (MAR./MAY, 2010)

ST 101 - PROBABILITY THEORY

(REPEAT)

Answer all questions

Time: Three hours

1. In an attempt to economize on her telephone bill, Rani times her calls and ensures that they never last longer than four minutes. The length of calls, T minutes, may be regarded as a random variable with probability density function:

$$f(t) = \begin{cases} kt & \text{if } 0 < t \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

- Find the value of k .
 - Find the mean and standard deviation of T .
 - Find the probability that a call lasts between three and four minutes.
 - What is the probability that, of five independent calls, exactly three last between three and four minutes.
2. (a) Show that if a random variable has the probability density function

$$f(x) = \frac{1}{2} e^{-|x|} \text{ for } -\infty < x < \infty$$

then its moment generating function is given by $M_x(t) = \frac{1}{1-t^2}$.

- (b) A random variable X has the density function

$$f(x) = \begin{cases} \frac{1}{2\sqrt{x}} & 0 < x < 1 \\ 0 & \text{elsewhere.} \end{cases}$$

Obtain the moment generating function and hence the mean and variance.

9

3. Tompkins Associates reports that the mean clear heights for a class A warehouse in the United States is 22 feet. Suppose that clear heights are normally distributed and that standard deviation is 4 feet. A class A warehouse in the United States is randomly selected.

- (a) What is the probability that the clear height is less than 17 feet?
- (b) What is the probability that the clear height is greater than 13 feet?
- (c) What is probability that the clear height is between 25 and 31 feet?
- (d) Suppose that the mean clear height of all United States class A warehouse is unknown. But the standard deviation is known to be 4 feet. What is the value of the mean clear height if 29% of United States class A warehouses have a clear height less than 20 feet?

4. (a) i. Prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

ii. Derive an equation for union of three elements.

iii. A problem in statistics is given to the three students A, B and C whose chance of solving it are 0.5, 0.75 and 0.25 respectively. What is the probability that the problem will be solved if all of them try independently?

(b) i. State and prove Bay's theorem

ii. Two methods A and B are available for teaching a certain industrial skill. The failure rate is 20% for A and 10% for B . However, B is more expensive and hence is used only 30% (A is used the other 70%). A worker was taught the skill by one of the methods but failed to learn correctly. What is the probability that the worker was taught by method A ?

5. (a) Data files on computers have sizes measured in megabytes. When files are sent from one computer to another down a communication link, the number of errors has a Poisson distribution. On average, there is one error for every 10 megabytes of data. Find the probability that a 2 megabyte file is transmitted:

i. Without error.

ii. With one or more errors.

(b) Suppose that the number of telephone calls coming into a telephone exchange between 10.00 am and 11.00 am, say X is a random variable with Poisson distribution with parameter 2. Similarly number of telephone calls arriving into the telephone

exchange between 11.00 am to 12.00 noon, say Y has a Poisson distribution with parameter 6. If X and Y are independent, what is the probability that more than 5 calls come in between 10.00 am and 12.00 noon?

6. (a) Among the 300 employees of a company 240 are union members, where as the others are not. If 6 of the employees are chosen by lot to serve on a committee that administers the pension fund, find the probability that 4 of the 6 will be union members using
- the formula for the hypergeometric distribution
 - the binomial distribution as an approximation.
- (b) A taxi company has 12 Ambassadors and 8 fiats. If 5 of these taxi cabs are in the shop for repairs and ambassador is as likely to be in for repairs as a fiat, what is the probability that
- 3 of them are ambassadors and 2 are fiats?
 - all of them are of the same make?

