EASTERN UNIVERSITY, SRI LANKA
EXTERNAL DEGREE
THIRD EXAMINATION IN SCIENCE - 2008/2009
SECOND SEMESTER (Feb./Apr., 2015)
EXTMT 310 - FLUID MECHANICS
(PROPER / REPEAT)

Answer all Questions **Time: Two hours**

Q1. (a) With the usual notation, derive the continuity equation for a fluid flow in the form

$$\frac{\partial \rho}{\partial t} + \underline{\nabla}.(\rho \underline{v}) = 0.$$

(b) If the fluid is incompressible, then deduce that the above equation takes the form

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$

in cartesian coordinates, where u, v and w are the cartesian components of the velocity.

Show that

$$u = \frac{kx}{(x^2+y^2+z^2)^{3/2}}, \quad v = \frac{ky}{(x^2+y^2+z^2)^{3/2}}, \quad w = \frac{kz}{(x^2+y^2+z^2)^{3/2}}$$

are the possible velocity components of an incompressible fluid flow.

Q2. (a) With the usual notation, derive the Euler's equation for an incompressible and inviscid fluid flow.

Hence show that the Euler's equation can be written as

$$(\underline{v}.\underline{\nabla})\underline{v} = \underline{F} - \frac{1}{\rho}\underline{\nabla} p,$$

for steady flow.

(b) An incompressible and inviscid fluid obeying Boyle's law $p = k\rho$, where k is a constant, is in motion in a uniform tube of small section. Prove that if ρ be the density of the fluid, then the velocity v at a distance x at time t in the tube is given by the equation

$$\frac{\partial^2 \rho}{\partial t^2} = \frac{\partial^2}{\partial x^2}[(v^2 + k)\rho].$$

Q3. (a) State and prove the Milne-Thomson Theorem.

Consider a stream with the complex potential $\omega = Uz$. When a circular cylinder is inserted, find the velocity potential and stream function. Also, prove that the greatest complex velocity of the motion is $2U$. Hence, Show that

$$2(\rho - \pi) = \rho U^2(1 - 4\sin^2\theta),$$

where U and π are the velocity and pressure, respectively, at infinity.

(b) Define Source, Sink and Strength of a source.

Two sources, each of strength m are placed at the points $(-a, 0), (a, 0)$ and a sink of strength $2m$ at the origin. Show that the stream lines are the curves $(x^2 + y^2)^2 = a^2(x^2 - y^2 + \lambda xy)$ where λ is a variable parameter.

Q4. Prove the following:

(a) If S is the boundary of a spherical surface lying wholly within the fluid, then the mean value φ of the velocity potential ϕ is equal to its value at the center of the sphere.

(b) If $\sum$ is the solid boundary of a large spherical surface of radius r, containing fluid in motion and also enclosing one or more closed surfaces, then the mean value φ of ϕ on $\sum$ is of the form

$$\varphi = \frac{m}{r} + c,$$

where m and c are constants, provided that the fluid extends to infinity and is at rest there.

(c) If the fluid is at rest at infinity and either ϕ or $\dfrac{\partial \phi}{\partial n}$ is described on each surface S_m, then ϕ is determined uniquely throughout V with an arbitrary constant.

(Hint:If the fluid is at rest at infinity and each surface S_m is rigid, then the kinetic energy of the moving fluid is given by

$$T = \frac{1}{2}\rho \int_v q^2 dv = \frac{1}{2}\rho \sum_{m=1}^{k} \int_{S_m} \phi \frac{\partial \phi}{\partial n} ds$$

here, the normal at each surface element dS being drawn outwards from the fluid.)