EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

FIRST EXAMINATION IN SCIENCE - 2015/2016

FIRST SEMESTER (July/ August, 2017)

PM 101 - FOUNDATION OF MATHEMATICS

Answer all questions | Time : Three hours

1. (a) Let p and q be two statements such that $\sim p \rightarrow q$ is false. Find the truth value of the following statements:

   i. $(p\vee \sim q) \rightarrow \sim p)$;

   ii. $\sim q \wedge (p \vee q)$.

   (b) Prove the following equivalences using the laws of logic:

   i. $(\sim p \vee q) \wedge (p\wedge \sim q) \equiv F$;

   ii. $[p \vee (q \wedge r)] \vee \sim [(\sim q \wedge \sim r) \vee r] \equiv p \vee q$,

   where p, q and r are statements.

   (c) Using the valid argument forms, draw a valid conclusion to the premises given below:

$$\sim p \vee q \longrightarrow r$$

$$\sim q \vee s$$

$$\sim t$$

$$p \rightarrow t$$

$$\sim p \wedge r \rightarrow \sim s,$$

   where p, q, r, s and t are statements.

2. (a) Let A, B, C be subsets of a set X. Prove the following:

   i. $A \Delta B = (A \cup B) \setminus (A \cap B)$;

   ii. $A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C)$;

   iii. $(A \cap B') \cup (A' \cap B) = A \cup B$ if and only if $A \cap B = \phi$.

   (b) For any set A, B and C, prove that $A \times (B \setminus C) = (A \times B) \setminus (A \times C)$.

3. (a) Let ρ be a relation defined on $\mathbb{C} \setminus \{0\}$ by $z_1 \rho z_2 \iff |z_1|(|z_2|^2 + 1) = |z_2|(|z_1|^2 + 1)$.

   Prove that ρ is an equivalence relation .

   If $a \in \mathbb{R}$ is such that $0 < a < 1$, sketch equivalence class of a, $[a]$, on the Argand diagram.

   (b) Let R be an equivalence relation on a set A. Prove the following:

   i. $[a] \neq \Phi$ for all $a \in A$,

   ii. $aRb \iff [a] = [b]$,

   iii. either $[a] = [b]$ or $[a] \cap [b] = \Phi$ for all $a \in A$.

4. (a) Define the following terms:

   i. *injective mapping*, ii. *surjective mapping*, iii. *inverse mapping*.

   (b) i. Let a function $f : \mathbb{R} \longrightarrow \mathbb{R}$ be defined by $f(x) = x|x|$. Show that f is bijective function and determine f^{-1}.

   ii. Is the function $g : \mathbb{R} \longrightarrow \mathbb{R}$ defined by $g(x) = x^2|x|$ a bijection? Justify your answer.

5. (a) Let $f : X \to Y$ be a mapping and A and B be any subsets of X. Prove the following:

   i. f is injective if and only if $f(A \cap B) = f(A) \cap f(B)$;

   ii. f is surjective if and only if $Y \setminus f(A) \subseteq f(X \setminus A)$.

   (b) If A and B are two countable sets then prove that $A \cup B$ is countable.

6. (a) State *division algorithm*.

   For any integer a , Prove that:

   i. $3 \mid a(a+1)(a+2)$;

   ii. $3 \mid a(2a^2 + 7)$;

   iii. if a is odd then $8 \mid (a^2 - 1)$.

(b) Using the Euclidean algorithm find the gcd(1819, 3587) and hence express it as a linear combination of 1819 and 3587.

(c) A grocer orders apples and bananas at a total cost Rs 840. If the apples cost Rs 25 each and the bananas Rs 5 each, how many of each type of fruit did he order?