EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

FIRST EXAMINATION IN SCIENCE - 2015/2016

SECOND SEMESTER (MAY/JUNE, 2018)

PM 107 - THEORY OF SERIES

Answer all question. Time: Two hours

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1. (a) Define the *convergence* and *divergence* of an infinite series $\sum_{n=1}^{\infty} a_n$. [10 marks]

   (b) Determine the convergence of the series $\sum_{n=1}^{\infty} a_n$ if a_n does not tend to zero as n goes to infinity. [30 marks]

   (c) Show that the series $\sum_{n=1}^{\infty}(k_1 a_n + k_2 b_n)$ is convergent, where $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ are two convergent series of non-negative real numbers and k_1, k_2 are two positive real numbers. [30 marks]

   (d) State the *comparison test* for convergence and divergence of an infinite series and use it to check the convergence of the series,
   $$\sum_{n=1}^{\infty} \frac{2^{(\frac{-1}{n})}}{n^3}.$$ [30 marks]

2. (a) Define the *absolute convergence* and *conditional convergence* of an infinite series $\sum_{n=1}^{\infty} a_n$. [10 marks]

   (b) Discuss the nature of convergence of the series,
   $$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{ln(n)}{n}.$$ [30 marks]

[P. T. O.]

[Question 2. continued...]

(c) State the *D'Alembert's ratio test* and use it to test the convergence of the series,

$$\sum_{n=1}^{\infty} \frac{n^2+2n+1}{3^n+2}.$$

[30 marks]

(d) Discuss the convergence of the series,

$$\sum_{n=1}^{\infty} \frac{(-1)^n(n^n)}{3^{(n^3+1)}},$$

by using the n^{th}-root test. [30 marks]

3. (a) Let the function $f(x)$ be positive, decreasing, and continuous on $[1,\infty)$. Show that the sequence,

$$(I_n - S_n),$$

is increasing and bounded above, where $I_n = \int_1^n f(x)dx$ and $S_n = \sum_{k=1}^{n} f(k)$.

[30 marks]

(b) State the *integral test* and use it to determine the convergence of the series,

$$\sum_{n=1}^{\infty} \frac{2}{3n+5}.$$

[35 marks]

(c) By using the *alternating series test*, comment on the convergence of the series,

$$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+1}.$$

[35 marks]

4. (a) Define a power series in x about point c with coefficients a_ns.

[10 marks]

(b) Find the interval of convergence and radius of convergence of the power series,

$$\sum_{n=1}^{\infty} \frac{(-1)^n n(x+3)^n}{4^n}.$$

[30 marks]

(c) Show that the function $f(x)$, which has derivatives of all order, can be expressed in a Taylor series,

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(c)}{n!}(x-c)^n,$$

where $f^{(n)}(x)$ is the n^{th}-derivatives of $f(x)$. [30 marks]

(d) Expand the function, $\ln[z(z+1)]$, in a Taylor series about $z = z_0$ up to order 3.

[30 marks]