EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

EXTERNAL DEGREE EXAMINATION IN SCIENCE (2010/2011)

SECOND YEAR SECOND SEMESTER (Apr./ May, 2017)

EXTMT 202 - METRIC SPACE

Special Repeat

Answer all questions **Time: Two hours**

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1. Define the terms

   - metric space.
   - complete metric space.

   (a) Let $X = \mathbb{R}$ and define $d : X \times X \to \mathbb{R}$ by

   $$d(x, y) = |x - y|, \quad \forall x, y \in \mathbb{R}$$

   Sow that (X, d) is a metric space.

   (b) Prove that every open ball is an open set.

   (c) Prove that, for any subset A of a metric space its interior A° is the largest open set contained in A.

   (d) Show that arbitrary intersection of closed set is closed.

2. (a) Let (X, d) be a metric space. Show that there is no sequence that can converges to two different limits.

   (b) Let (X, d) be a metric space. Show that if a sequence $\{x_n\}$ converges in x then it is bounded.

(c) Let (Y, d) be a subspace of a metric space (X, d). Prove that $A \subseteq Y$ is open in Y and only if there exist a set G open in$X$ such that $A = Y \cap G$.

(d) In a metric space, prove that any Cauchy sequence that contains a convergent sub sequence is convergent.

3. Define the following terms in a metric space:

- separated sets;
- disconnected set;
- connected sets.

(a) Prove that two open subsets of a metric space are separated if and only if they ar disjoint.

(b) Suppose that an open set G is the union of two separated sets A and B in a metri space (X, d). Prove that A and B are open.

(c) Prove that a metric space (X, d) is disconnected if and only if it can be written as a union of two non-empty disjoint open sets.

4. (a) Define the term compact subset of a metric space.

   i. Show that every finite subset of a metric space is compact.

   ii. Prove that every compact subset of a metric space is is bounded.

(b) Let (X, d_1) and (Y, d_2) be a metric spaces and let $f : X \to\to Y$ be a function. Prove that the following:

   i. f is continuous if and only if $f^{-1}(G)$ is open in X whenever G is open in Y.

   ii. f is continuous if and only if $f^{-1}(B^{\circ}) \subseteq (f^{-1}(B))^{\circ}, \ \forall B \subseteq Y$