



EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

EXTERNAL DEGREE EXAMINATION IN SCIENCE 2010/2011

SECOND YEAR SECOND SEMESTER (Apr./May, 2017)

EXTMT 205 - DIFFERENTIAL GEOMETRY

(Special Repeat)

Answer all questions

Time : One hour

1. (a) State Serret-Frenet formula.

Let  $\Gamma$  be a curve with constant torsion  $\tau$  and a point  $Q$ , a constant distance  $c$  from the point  $P$  on  $\Gamma$ , on the binormal to the curve  $\Gamma$  at  $P$ . Show that the angle between the binormal to the locus of  $Q$  and the binormal to the given curve  $\Gamma$  is

$$\tan^{-1} \left( \frac{c\tau^2}{\kappa\sqrt{1+c^2\tau^2}} \right)$$

where  $\kappa$  is the curvature of the curve  $\Gamma$  at  $P$ .

- (b) Define "normal plane" of a space curve.

Find the equation for the normal to the curve  $\underline{r} = (1+t)\underline{e}_1 - t^2\underline{e}_2 + (1+t^3)\underline{e}_3$  at the point  $t = 1$ .

2. What is meant by saying that a curve is a helix?

- (a) Prove that, a space curve to be a helix if and only if  $\frac{\tau}{\kappa}$  is constant, where  $\tau$  and  $\kappa$  are torsion and curvature of the given space curve, respectively.

- (b) Show that the curve given by  $x = a \cos \theta$ ,  $y = a \sin \theta$  and  $z = a\theta \cot \beta$  is a helix, where  $a$  and  $\beta$  are constants.