EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

EXTERNAL DEGREE EXAMINATION IN SCIENCE 2010/2011

SECOND YEAR SECOND SEMESTER (Apr./May, 2017)

EXTMT 217 - MATHEMATICAL MODELING

(REPEAT)

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swer all questions **Time : Two hours**

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1. Describe the steps involved in a mathematical model building process.

   Give a mathematical formulation for the following problem:

   Milk can be consumed either directly or in processed forms such as butter, yoghurt and cheese etc. The company X is the sole producer of milk in a certain region and also owns the factory to process milk into various forms. The problem for the company is to determine the relative quantities of the various products necessary to achieve this goal.

2. State the Newton's law of cooling.

   Mary puts a can of soda into a refrigerator at 1.00 PM that has been sitting in the room temperature 70°F. The temperature in the refrigerator is 40°F. Fifteen minutes later, at 1.15 PM; the temperature of the soda has fallen to 60°F. At some time later, Mary removes the soda from the refrigerator to the room, where at 2.00 PM the temperature of the soda is 60°F. At what time did Mary removes the soda from the refrigerator?

3. Suppose a x force and a y force are engaged in combat. Let $x(t)$ and $y(t)$ denote tl respective strength of the forces at time t, when t is measured in days from the start of tl combat. Conventional combat model is given by

$$\begin{aligned} \frac{dx(t)}{dt} &= -a\,x(t) - b\,y(t) + P(t); \\ \frac{dy(t)}{dt} &= -d\,y(t) - c\,x(t) + Q(t). \end{aligned}$$

Explain the terms involved in these equations.

By using the assumptions that there is no reinforcement arrived and no operational loss occur, obtain a simplified model, and hence show that

$$x(t) = x_0 \cosh(\beta t) - \gamma y_0 \sinh(\beta t),$$

where $\beta = \sqrt{bc}$, $\gamma = \sqrt{\dfrac{b}{c}}$ and x_0, y_0 are the initial strength of the respective forces.

4. The fish population in a certain part of the sea can be separated into prey population(fo fish) $x(t)$ and predator population(Selachians) $y(t)$. The model governing the interaction the selachians and food fish in the absence of fishing is given by

$$\begin{aligned} \frac{dx}{dt} &= ax - bxy; \\ \frac{dy}{dt} &= -cx + dxy. \end{aligned}$$

Explain the terms involved in this model.

Show that $\dfrac{y^a}{e^{by}} \cdot \dfrac{x^c}{e^{dx}} = k$, where k is a constant.

Let $x(t)$ and $y(t)$ be the periodic solution of the above equations.

If $\overline{x} = \dfrac{1}{T}\displaystyle\int_0^T x(t)\,dt$ and $\overline{y} = \dfrac{1}{T}\int_0^T y(t)\,dt$ then show that $\overline{x} = \dfrac{c}{d}$ and $\overline{y} = \dfrac{a}{b}$ where T is t period.

Hence show that a moderate amount of fishing increases the average number of food f and decreases the average number of selachians.