EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS
SECOND EXAMINATION IN SCIENCE - 2009/2010
SECOND SEMESTER (April/May, 2012)
MT 205 - DIFFERENTIAL GEOMETRY
(PROPER & REPEAT)

Answer all Questions — Time: One hour

Q1. What is meant by saying that a curve is a helix?

Prove, with the usual notations, that the necessary and sufficient condition for a curve to be a helix is that $\frac{\tau}{\kappa}$ is constant, where τ is a torsion and κ is a curvature of the curve.

Show that the curve $x = a\cos\theta$, $y = a\sin\theta$ and $z = a\theta\cot\beta$, is a helix, where a and β are constants.

Q2. Define the term osculating sphere of a space curve and find its radius and center.

Show that the tangent, principle normal and binormal to the locus C_1 of the center of the osculating sphere of a given curve C are parallel to the binormal, principle normal and tangent to C, respectively at the corresponding points.

Deduce, with the usual notations that

$$\frac{\rho}{\sigma} + \frac{d}{ds}\left(\frac{\rho'}{\tau}\right) = 0$$

is the necessary and sufficient condition for a curve to lie on a sphere. Here ρ is the radius of curvature, σ is the radius of torsion, τ is a torsion and s is the arc length of the curve.