EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE - 2009/2010

SECOND SEMESTER (PROPER/REPEAT)

(April 2012)

PH 206 WAVES AND VIBRATIONS

Time: 01 hour.

Answer ALL Questions

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1. Equation of motion of a driven harmonic oscillator is given by

$$m\frac{d^2x}{dt^2} + b\frac{dx}{dt} + kx = F_0\cos\omega t,$$

where the symbols referring to their usual meanings.

(a) Show that at steady state the displacement is given by

$$x = A(\omega)\cos(\omega t - \phi),$$

in terms of the amplitude of oscillation

$$A(\omega) = \frac{F_0}{m\sqrt{({\omega_0}^2 - \omega^2)^2 + (2\gamma\omega)^2}}\ ;\ \omega_0 = \sqrt{\frac{k}{m}};\ \gamma = \frac{b}{2m}\ ,\ \text{and}$$

the phase angle

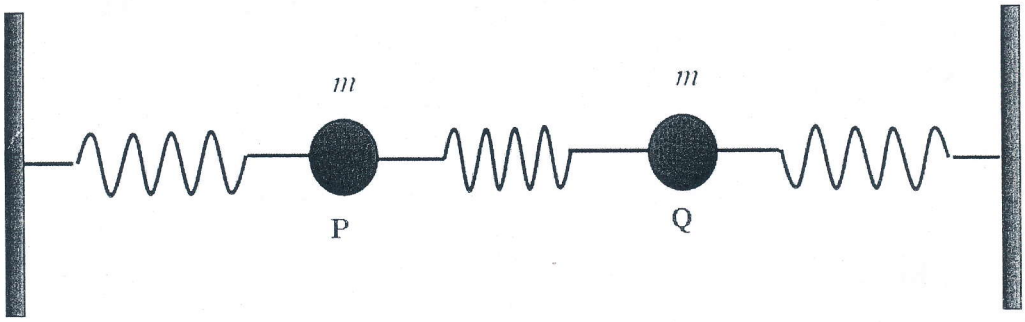
$$\phi = \tan^{-1}\left(\frac{2\gamma\omega}{{\omega_0}^2 - \omega^2}\right).$$

(b) Show that the resonance occurs at the frequency of the applied force F_0 at

$$f = \frac{\sqrt{{\omega_0}^2 - 2\gamma^2}}{2\pi}.$$

(c) Sketch the variation of $A(\omega)$ and ϕ as a function of angular frequency ω for differen values of γ.

2. Two bobs P and Q, each of mass m are connected by three springs of length l, as shown in figure. The bobs P and Q undergo a transverse vibration with vertical displacements y_1 and respectively.



(a) Assuming that the tension on the springs are T, show that the equation of motion of P Q are given by

$$m\frac{d^2y_1}{dt^2} + \frac{2T}{l}y_1 - \frac{T}{l}y_2 = 0 \quad \text{and}$$

$$m\frac{d^2y_2}{dt^2} + \frac{2T}{l}y_2 - \frac{T}{l}y_1 = 0.$$

State clearly the assumptions that you made in the derivations.

(b) Show that the angular frequencies of the normal modes of P and Q are given by

$$\omega_1 = \sqrt{\frac{T}{ml}} \quad \text{and} \quad \omega_2 = \sqrt{\frac{3T}{ml}}.$$

Hence, discuss the vibrations of the system when the normal mode frequencies are eq to ω_1 and ω_2.