EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE - 2009/2010

SECOND SEMESTER (PROPER/REPEAT)

(April 2012)

PH 207 ELECTRICITY AND MAGNETISM II

Time: 01 hour.

Answer ALL Questions

You may find the following information useful.

- The vector identities

$$\vec{\nabla}.(\vec{A} \times \vec{B}) = \vec{B}.(\vec{\nabla} \times \vec{A}) - \vec{A}.(\vec{\nabla} \times \vec{B})$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = \vec{\nabla}.(\vec{\nabla}.\vec{A}) - \nabla^2 \vec{A}$$

- Gauss divergence theorem

$$\oint_S \vec{A}.\overrightarrow{da} = \int_V \vec{\nabla}.\vec{A}d\tau$$

- Strokes theorem

$$\oint_C \vec{A}.\vec{dl} = \int_S (\vec{\nabla} \times \vec{A})\overrightarrow{da}$$

The symbols have their usual meanings.

1. (a) Show how the equation $\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$ may be derived on the basis of Gauss's law in electrostatics.

(b) State Biot-Savart law and hence derive the equation $\vec{\nabla}.\vec{B} = 0$.

(c) State Ampere's circuital law and hence derive the equation $\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}$. Hence derive the equation $\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right)$ by incorporating Maxwell's assumptions.

(d) Using the concept of Faraday's law of electromagnetic induction, derive the equation $\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$.

Here the symbols have their usual meanings and assume that the medium is free space.

2. Write down Maxwell's equations in free space.

(a) Starting from appropriate Maxwell's equation obtain the wave equations for electric field $\vec{E}$ and magnetic field $\vec{B}$ in free space and show that the velocity of the electromagnetic wave is given by

$$v = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$$

(b) An electromagnetic wave propagating along z-axis in free space is described by $\vec{E} = \hat{x} E_0 e^{i(\omega t - kz)}$ and $\vec{B} = \hat{y} B_0 e^{i(\omega t - kz)}$, where the symbols have their usual meanings. Using an appropriate Maxwell's equation, show that

$$\frac{E_0}{B_0} = \frac{\omega}{k}$$