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EASTERN UNIVERSITY, SRI LANKA
FACULTY OF COMMERCE AND MANAGEMENT
FIRST YEAR – SECOND SEMESTER EXAMINATION IN
BUSINESS ADMINISTRATION / COMMERCE 2007 / 2008 (SEPT. 2009)
COM 1032 BASIC CALCULUS

Answer all Questions

Time: 03 hours

- (i) Specify the domain of each of the following functions:

a) $f(x) = \frac{x-3}{x^2+x-2}$

b) $f(x) = \sqrt{x^2 - 9}$

- (ii) a) Find $f(x^2 + 1)$ if $f(x) = \sqrt{x} + \frac{2}{x-1}$

- b) Find the composite function $g[h(x)]$ if $g(u) = u^2 + 2u + 1$ and $h(x) = 1 - x$

- (iii) A Bus company's revenue as a function of the number of passengers is given below.

$$R(x) = \begin{cases} 2400, & \text{if } 0 < x \leq 40 \\ 80x - 0.5x^2, & \text{if } 40 < x < 80 \\ 40x, & \text{if } 80 \leq x \end{cases}$$

- (iv) a) Sketch the graph of the function.

Use the graph to answer the following questions.

- b) Is $R(x)$ continuous?

- c) Over what intervals is $R(x)$ increasing?

- d) Over what intervals is $R(x)$ a linear function? quadratic function?

(20 Marks)

- (i) Evaluate the following limits:

a) $\lim_{x \rightarrow 3} \frac{x^2+2x-15}{x^2-9}$

b) $\lim_{x \rightarrow 0} \frac{\sqrt{x-1} - 1}{x}$

- (ii) Find $\frac{dy}{dx}$ for the functions given below

a) $y = e^{x \ln x^2}$

b) $y = \frac{3x+1}{\sqrt{1-4x}}$

- (iii) a) Given $e^{x+y} = xy$, use implicit differentiation to find $\frac{dy}{dx}$. Express your answer in terms of x and y and simplify.

b) if $x = \frac{2t}{1+t^2}$ and $y = \frac{1-t^2}{1+t^2}$, Show that $\frac{dy}{dx} + \frac{x}{y} = 0$

c) Find the fourth derivative of the function $y = x^5 - 2x^4 + x^3 - 3x^2 + 5x - 1$

d) If $2y^2 - 5x^2 = 3$ find $\frac{d^2y}{dx^2}$ in terms of x and y . Simplify your answer.

(20 Marks)

3. (i) Suppose the demand q and price p for a certain commodity are related by the linear function $Q = 240 - 2p$.

a) Express the elasticity of demand as a function of p .

b) Calculate the elasticity of demand when the price is 100. Interpret your answer.

c) At what price is the elasticity of demand equal to -1? What is the economic significance of this price?

(ii) Suppose the total cost of manufacturing q units of a certain commodity is $C(q) = 3q^2 + q + 48$.

a) At what level of production is the average cost per unit the smallest?

b) At what level of production is the average cost per unit equal to the marginal cost?

c) On the same set of axes, graph the average cost and marginal cost functions.

(iii) Given the demand function $p = 8.25e^{-0.02Q}$, determine the quantity and price at which revenue will be maximized.

4. (i) a) State both a necessary condition and a sufficient condition for $z = f(x, y)$ to have

(i) A relative minimum

(ii) A relative maximum

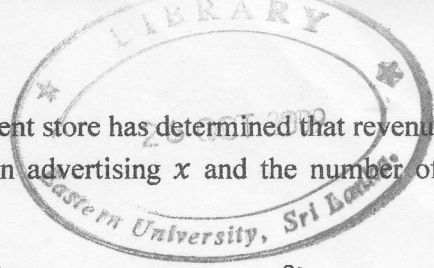
(iii) A saddle point

b) Find the relative minima, relative maxima and saddle points in the following function

$$f(x, y) = 3x^3 + 1.5y^2 - 18xy + 17$$

(ii) a) A dairy produces whole milk and skim milk in quantities x and y gallons, respectively. Suppose that the price of whole milk is $p(x) = 100 - x$ and that of skim milk is $q(y) = 100 - y$ and assume that $C(x, y) = x^2 + xy + y^2$ is the joint cost function for the commodities. What should x and y be in order to maximize profit?

b) The number of units of a product that are manufactured by a company is given by $f(l, k) = 800l^{2/3}k^{1/3}$, where l is the units of labour and k is the units of capital. Find the marginal productivity of labour and the marginal productivity of capital. The company is currently utilizing 27 units of labour and 64 units of capital. Interpret the results.



- (iii) The marketing manager of a department store has determined that revenue, in rupees is related to the number of units of television advertising x and the number of units of newspaper advertising y by the function

$$R(x, y) = 200(20x + 5y + 6xy - x^2).$$

Each unit of television advertising cost Rs. 3000 and each unit of newspaper advertising cost Rs. 1500. If the advertising budget is Rs. 30000, find the maximum revenue.

(20 Marks)

- (i) Evaluate the following:

(a) $\int (2x + 3) \sqrt{x^2 + 3x - 1} dx$

(b) $\int \frac{x}{e^x} dx$

(c) $\int_1^2 \frac{2x}{1+x^2} dx$

(d) $\int_0^4 \sqrt{3x + 4} dx$

- (ii) ABC co. Ltd. has approximated the marginal revenue function for one of its product by $MR = 20x - 2x^2$. The marginal cost function is approximated by $MC = 81 - 16x + x^2$. Find the maximum profit.

- (iii) The demand and supply function under perfect competition are $P_d = 16 - x^2$ and $P_s = 2x^2 + 4$ respectively. Find the market price, consumer's surplus and producer's surplus.

(20 Marks)