EASTERN UNIVERSITY, SRI LANKA

THIRD EXAMINATION IN SCIENCE - 2015/2016

SECOND SEMESTER (Oct./Nov., 2018)

AM 310 - FLUID MECHANICS

Answer all questions Time : Two hours

1. (a) Let the velocity of a fluid flow be $\underline{V} = (1 + At)\underline{i} + x\underline{j}$, where A is a constant.

    i. Find the equation of streamline passing through the point (x_0, y_0) at $t = t_0$.

    ii. Obtain the equation of path line of a fluid element which comes to (x_0, y_0) at $t = t_0$.

    Hence, show that the streamline and path line coincide when $A = 0$.

   (b) An inviscid fluid moves through a straight long tube of narrow core. The density of the fluid at a distance x from one end of the tube at time t is $\rho_0 e^{a(x-ct)}$, where a, c and ρ_0 are constants. If the velocity of the fluid at the end, $x = 0$, is equal u then show that the velocity of the fluid at a distance l from that end is $c + (u - c)e^{-al}$.

2. (a) With the usual notation, derive the *Euler's equation* for an incompressible and inviscid fluid flow.

   Hence show that if the fluid flow is steady the *Euler's equation* can be written as

$$(\underline{V}.\underline{\nabla})\underline{V} = \underline{F} - \frac{1}{\rho}\underline{\nabla}p.$$

   (b) An incompressible fluid contained in a cylindrical vessel which rotate with constant angular velocity ω about the vertical axis. If the gravity is the only external force acting on that fluid, find the surface of equal pressure in the liquid. If the fluid surface is open to the atmosphere, show that, with the suitable axis of coordinates, its equation may be written in the form

$$x^2 + y^2 = \frac{2g}{\omega^2} z.$$

3. Let a gas occupy the region $r \leq R$, where R is a function of time t, and a liquid of constant density ρ lie outside the gas. Assume that there is contact between the gas and the liquid all the time and the motion is symmetric about the origin $r = 0$. If the velocity at $r = R$, the gas liquid boundary is continuous then show that the pressure p at a point $P(\underline{r}, t)$ in the liquid is given by

$$\frac{p}{\rho} + \frac{1}{2}\left(\frac{R^2\dot{R}}{r^2}\right)^2 - \frac{1}{r}\frac{d}{dt}(R^2\dot{R}) = f(t), \text{ where } r = |\,\underline{r}\,|\,.$$

Further, if it is given that the liquid extends to infinity and is at rest with constant pressure Π at infinity, prove that the gas liquid interface pressure is equal to $\Pi + \dfrac{\rho}{2R^2}\dfrac{d}{dR}(R^3\dot{R}^2)$.

If the gas obeys the Boyle's law $pv^{4/3} =$ constant, where v is the volume of the gas, and expands from rest at $R = a$ to a position of rest $R = 2a$, show that the ratio of initial pressure of the gas to the pressure of the liquid at infinity is 14:3.

4. Write down the *Bernoulli's equation* for steady motion of an inviscid incompressible fluid.

Let a fluid of density ρ fill the region on the positive side of the x axis and the axis of y being a fixed boundary. If a two dimensional source of strength m is situated at the point $(c, 0)$, find the points on the boundary at which the velocity is maximum. *Show that the resultant thrust on the part of the axis of y which lies between* $y = \pm l$ is $2\Pi l - 2m^2\rho\left[\frac{1}{c}\tan^{-1}\left(\frac{l}{c}\right) - \frac{l}{c^2 + l^2}\right]$, *where Π is the pressure at* infinity.