EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

EXTERNAL DEGREE EXAMINATION IN SCIENCE (2010/2011)

FIRST YEAR FIRST SEMESTER (Apr./ May, 2017)

EXTMT 106 - TENSOR CALCULUS

Special Repeat

Answer all questions **Time: One hour**

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1. (a) Define the Covariant tensor A_{pq} and the Contravariant tensor A^{pq}.

   (b) Write down the law of transformation for the following tensors:

   i. A^{qr}_{ms};

   ii. B^{ijk}_{lm};

   iii. C_{mn}.

   (c) If $ds^2 = g_{jk}\, dx^j\, dx^k$ is an invariant, show that g_{jk} is a symmetric covariant tensor of rank two.

   (d) Find g and g^{jk} corresponding to the line element

   $$ds^2 = 5(dx^1)^2 + 3(dx^2)^2 + 4(dx^3)^2 - 6dx^1dx^2 + 4dx^2dx^3.$$

2. (a) Define the Christoffel's symbols of the first and second kind.

   (b) Determine the Christoffel's symbols of the second kind for the line element given by

   $$ds^2 = dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta\, d\phi^2.$$

   (c) With the usual notations, prove the followings:

   i. $\dfrac{\partial g_{pq}}{\partial x^m} = [pm, q] + [qm, p]$;

   ii. $[pq, r] = g_{rs}\, \Gamma^s_{pq}$;

   iii. $\dfrac{\partial g^{pq}}{\partial x^m} = -g^{pn}\Gamma^q_{mn} - g^{qn}\Gamma^p_{mn}$.