EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

THIRD EXAMINATION IN SCIENCE -2009/2010

FIRST SEMESTER (Feb./ March, 2013)

MT 306 – PROBABILITY THEORY

(SPECIAL REPEAT)

Time: Two hours.

Answer **all** questions.
Statistical tables will be provided.

01. (a) State and prove the **Total Probability** theorem and **Bayes'** theorem.

(b) Students in a school wear ties in four different colors red, blue, green and yellow according to their sport meet houses. Composition of students wear ties of different colors is same (25% each). Respective probabilities, of the each group to win a certain game, are 40%, 30%, 20% and 10%. A student was selected at random and found to be a student won the game. What is the probability that the selected student is a student wears red tie.

02. The weight of milk packets manufactured by a certain company is normally distributed with mean of 400g and standard deviation of 10g. If a milk packet is selected randomly from this company, find the probability that weight of the selected packet is:

(i) greater than 420g;

(ii) less than 360g;

(iii) between 370g and 420g.

(iv) How many packets will have more than 360g of weight, if 500 packets are taken from this company?

(v) What is the maximum weight that 95% of the packets manufactured will have?

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03. Continuous random variables X and Y have the following joint probability density function

$$f_{XY}(x,y) = \begin{cases} c\left(x^2 + \frac{1}{2}xy\right) & , \text{ if } 0 < x < 1, 0 < y < 2, \\ 0 & , \text{ otherwise}. \end{cases}$$

where c is a constant.

Find the followings:

(i) Value of c ;

(ii) The cumulative joint probability distribution function $F_{XY}(x,y)$;

(iii) Marginal density function of X and Y, $f_X(x)$ and $f_Y(y)$, respectively;

(iv) $E(X)$ and $E(Y)$.

04. (a) Assume $X_1, X_2, X_3, \ldots\ldots, X_n$ be a random sample from a Poisson distribution with parameter λ. Find an estimator for λ using method of moments. Use following data to estimate λ.

5, 9, 7, 8, 5, 5, 7, 6, 3, 8.

(b) Let $X_1, X_2, X_3, \ldots\ldots, X_n$ be a random sample from an Exponential distribution with parameter λ. Show that $\frac{1}{\overline{X}}$ is the maximum likelihood estimator for λ, where $\overline{X}$ is the sample mean.