EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS
THIRD YEAR EXAMINATION IN SCIENCE - 2010/2011
SECOND SEMESTER (June - 2014)
MT 307 - CLASSICAL MECHANICS
SPECIAL REPEAT

Answer all Questions **Time: Three hours**

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1. Two frames of reference S and S' have a common origin O, and S' rotates with constant angular velocity $\underline{\omega}$ relative to S. At a time t a particle P has position vector $\underline{r}$ referred to O; and $\dot{\underline{r}}$ and $\ddot{\underline{r}}$ denote the velocity and acceleration of P relative to S' respectively.

   Prove that the acceleration of P relative to S is

   $$\ddot{\underline{r}} + 2\underline{\omega} \wedge \dot{\underline{r}} + \underline{\omega} \wedge (\underline{\omega} \wedge \underline{r}).$$

   An object is thrown downward with an initial speed v_0. Prove that after time t the object is deflected east of the vertical by the amount

   $$\omega v_0 \sin\lambda t^2 + \frac{1}{3}\omega g \sin\lambda t^3,$$

   where λ is the earth's co - latitude.

2. (a) With the usual notations, obtain the equations of motion for a system of N particles in the following forms:

i. $M\underline{f}_G = \sum_{i=1}^{N} \underline{F}_i,$

ii. $\frac{d\underline{H}}{dt} = \sum_{i=1}^{N} \underline{r}_i \wedge \underline{F}_i,$

where $\sum_{i=1}^{N} \underline{h}_i = \underline{H}$ and $\underline{h}_i = \underline{r}_i \wedge m_i \underline{v}_i$.

(State clearly the results that you may use)

(b) A solid of mass M is in the form of a tetrahedron $OXYZ$, the edges OX, OY, OZ of which are mutually perpendicular, rests with XOY on a fixed smooth horizontal plane and YOZ against a smooth vertical wall. The normal to the rough face XYZ is in the direction of a unit vector $\underline{n}$. A heavy uniform sphere of mass m and center C rolls down the face causing the tetrahedron to acquire a velocity $-V\underline{j}$ where $\underline{j}$ is the unit vector along OY.

If $\overrightarrow{OC} = \underline{r}$, then prove that

$$(M+m)V - m\underline{\dot{r}} \cdot \underline{j} = \text{constant}$$

and that

$$\frac{7}{5}\,\underline{\ddot{r}} = \underline{f} - \underline{n}(\underline{n} \cdot \underline{f})\ ,$$

where $\underline{f} = \underline{g} + \dot{V}\underline{j}$ and $\underline{g}$ is the acceleration of gravity.

3. With the usual notation obtain the Euler's equations for the motion of the rigid body, having a point fixed, in the form:

$$A\dot{\omega}_1 - (B - C)\omega_2\omega_3 = N_1,$$

$$B\dot{\omega}_2 - (C - A)\omega_1\omega_3 = N_2,$$

$$C\dot{\omega}_3 - (A - B)\omega_1\omega_2 = N_3.$$

A body moves under no forces about a point O. The pricipal moment of inertia at O being $6A$, $3A$, A. Initially the angular velocity of the body has components $\omega_1 = n$, $\omega_2 = 0$, $\omega_3 = 3n$ about the principal axis. Show that at any latter time $\omega_2 = -\sqrt{5}n \tanh \sqrt{5}nt$.

4. Obtain the Lagrange's equations of motion using D'Alembert's principle for a conservative holonomic dynamical system.

   Use the Lagrangian method and obtain the equations of motion for a spherical pendulum of length r.

5. With the usual notations, derive Lagrange's equation for the impulsive motion from Lagrange's equations for a holonomic system in the following form

$$\triangle\left(\frac{\partial T}{\partial \dot{q}_j}\right) = S_j. \qquad j = 1, 2, ..., n.$$

   A uniform rod AB of length $2a$ and mass m has a particle of mass M attached to the end B. It is at rest on a smooth horizontal table when an impulse I is applied at A in a direction perpendicular to AB, and in the plane of the table. Find the initial velocities of A and B and prove that the resulting kinetic energy is

$$\frac{2I^2(m+3M)}{m(m+4M)}.$$

6. (a) Define the poisson bracket.

   Show that the Hamiltonian equations of the holonomic system may be written in the form

$$\dot{q}_k = [q_k, H], \qquad \dot{p}_k = [p_k, H],$$

   and show that for any function $f(q_i,\ p_i,\ t)$, $\dfrac{df}{dt} = \dfrac{\partial f}{\partial t} + [f,\ H]$.

   (b) Show that, if f and g are constants of motion then their poisson bracket $[f, g]$ is also a constant of motion.