EASTERN UNIVERSITY, SRI LANKA

THIRD EXAMINATION IN SCIENCE - 2010/2011

SECOND SEMESTER (March/April, 2014)

MT 310 - FLUID MECHANICS

SPECIAL REPEAT

Answer all questions | Time : Two hours

1. (a) Derive the continuity equation for a fluid flow in the form

$$\frac{D\rho}{Dt} + \rho \underline{\nabla} . \underline{q} = 0,$$

where ρ and $\underline{q}$ are the density and the velocity of the fluid.

Hence, establish the equation of continuity for an incompressible fluid in the form $\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$ in cartesian coordinates, where u, v and w are the cartesian components of the velocity.

(b) Show that $\frac{k}{r^5}\left(3x^2 - r^2, 3xy, 3xz\right)$, where $r^2 = x^2 + y^2 + z^2$ and k is a constant, represents the velocity field in a possible fluid motion.

Show also that this motion is irrotational and hence determine the streamlines.

2. Let a gas occupy the region $r \leq R$, where R is a function of time t, and a liquid of constant density ρ lie outside the gas. By assuming that there is contact between the gas and the liquid all the time and that the motion is symmetric about the origin $r = 0$, show that the motion is irrotational.

If the velocity at $r = R$, the gas liquid boundary is continuous then show that the pressure p at a point $P(\underline{r}, t)$ in the liquid is given by

$$\frac{p}{\rho} + \frac{1}{2}\left(\frac{R^2\dot{R}}{r^2}\right)^2 - \frac{1}{r}\frac{d}{dt}(R^2\dot{R}) = f(t), \text{ where} r = \mid \underline{r} \mid .$$

Further, if it is given that the liquid extends to infinity and is at rest with constant pressure Π at infinity, prove that the gas liquid interface pressure is equal to $\Pi + \frac{\rho}{2R^2}\frac{d}{dR}(R^3\dot{R}^2)$.

If the gas obeys the Boyle's law $pv^{4/3}$ = constant, where v is the volume of the gas, and expands from rest at $R = a$ to a position of rest $R = 2a$, show that the ratio of initial pressure of the gas to the pressure of the liquid at infinity is 14:3.

3. (a) Let a two dimensional source of strength m is situated at origin. Show that the complex potential w at a point $P(z)$ due to this source is given by $w = -m \ln z$.

   (b) Two sources, each of strength m placed at $(-a, 0), (a, 0)$ and a sink of strength $2m$ at the origin. Show that the streamlines are the curved

$$(x^2 + y^2)^2 = a^2(x^2 - y^2 + \lambda xy),$$

   where λ is a variable parameter.

   Show also that the fluid velocity at any point is $\frac{2ma^2}{r_1 r_2 r_3}$, where r_1, r_2 and r_3 are the distances of points from he sources and the sink.

4. Write down the Bernoulli's equation for steady motion of an inviscid incompressible fluid.

   A three dimensional doublet of strength μ whose axis is in the direction of $\overrightarrow{Ox}$ is distant a from a rigid plane $x = 0$ which is the sole boundary of liquid of density ρ, infinite in extent. Find the pressure at a point on the boundary distant r from the doublet. If the pressure at infinity is p_∞, then show that the pressure on the plane is least at a dstance $\frac{\sqrt{5}a}{2}$ from the doublet.