# EASTERN UNIVERSITY, SRI LANKA

## EXTERNAL DEGREE EXAMINATION IN SCIENCE - 2009/2010

## SECOND YEAR FIRST SEMESTER (June / Sept. , 2012)

## EXTMT 203 - EIGENSPACE AND QUADRATIC FORMS

## (PROPER & REPEAT)

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**Answer all Questions** **Time: Two hours**

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1. (a) Define the term eigenvalue and eigenvector of a linear transformation.
Find the eigenvalues and eigenvectors of the matrix

$$\begin{pmatrix} 1 & -1 & 1 \\ -1 & 1 & 1 \\ -1 & -1 & 3 \end{pmatrix}.$$

(b) i. Prove that eigenvectors that corresponding to distinct eigenvalues of a linear transformation $T : V \to V$ are linearly independent.

ii. Show that 0 is an eigenvalue of $T$ if and only if $T$ is singular.

iii. Suppose $\lambda$ is an eigenvalue of an invertible operator $T$. Show that $\lambda^{-1}$ is an eigenvalue of $T^{-1}$.

(c) Orthogonally diagonalize the matrix

$$A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}.$$

2. Define the term minimum polynomial of a square matrix.

(a) State the Cayley - Hamilton theorem.

Find the minimum polynomial of the square matrix

$$\begin{pmatrix} 2 & 1 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -2 & 4 \end{pmatrix}.$$

(b) Prove that for any square matrix $A$, the minimum polynomial ex
unique.

(c) Let $M = \begin{pmatrix} A & 0 \\ 0 & B \end{pmatrix}$, where $A$ and $B$ are square matrices. Sho
minimum polynomial $m(t)$ of $M$ is the least common multiple of th
polynomials $g(t)$ and $h(t)$ of $A$ and $B$ respectively.

3. (a) Find an orthogonal transformation which reduces the following qua
to a diagonal form

$$5x_1^2 + 6x_2^2 + 7x_3^2 - 4x_1x_2 + 4x_2x_3 = 1.$$

(b) Simultaneously diagonalize the following pair of quadratic forms

$$\phi_1 = x_1^2 - x_2^2 - 2x_3^2 - 2x_1x_2 + 4x_2x_3,$$

$$\phi_2 = x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 - 2x_2x_3.$$

4. (a) What is meant by an inner product on a vector space.

Let $x = (x_1, x_2, ..., x_n)$, $y = (y_1, y_2, ..., y_n) \in \mathbb{R}^n$, where $x_i, y_i \in \mathbb{R}$,

Let the inner product $< ., . >$ be defined on $\mathbb{R}^n$ as

$$< x, y >= xy^T = \sum_{i=1}^{n} x_i y_i.$$

Show that $(\mathbb{R}^n, < ., . >)$ is an inner product space.

(b) State and prove Cauchy Schwarz Inequality.

(c) State the Gram Schmidt Process.

Find the orthonormal set for span of $M$ in $\mathbb{R}^4$, where

$$M = \{(1, 0, -1, 0)^T, (0, 1, 2, 1)^T, (2, 1, -1, 0)^T\}.$$

Second

Eastern University, Sri Lanka

Year Second Semester Examination in Science – 2008/2009

External Degree Programmee (Re-repeat)

EXTBT-204 / Plant Structure & Systematica (Feb/Mar-2012)



Answer all questions

Time: Two hours

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1. Briefly write on the followings
   (a) Dispersal of fruits by water
   (b) Reproductive structure of an angiosperm
   (c) Modifications of axillary-bud of an angiosperm

2. Compare the followings
   (a) Qualitative and quantitative characters
   (b) Tap root and fibrous root
   (c) True fruits and false fruits

3. Briefly write the importance of the followings
   (a) Herbarium
   (b) Type specimens
   (c) Keys in plant systematic

4. Write short notes on the followings
   (a) Placentation
   (b) Advantages and disadvantages of scientific names
   (c) Natural classification