EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS
EXTERNAL DEGREE EXAMINATION IN SCIENCE
THIRD YEAR FIRST SEMESTER - 2009/2010
(May/Sept., 2012)
EXTMT 304 - GENERAL TOPOLOGY
(PROPER & REPEAT)

Answer all Questions — Time: Two hours

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1. (a) Define the following terms:

   i. Topology on a set;

   ii. Subspace of a topological space;

   iii. Neighborhood of a point.

   (b) Prove that the intersection of two topologies of a set X is again a topology.

   (c) i. Let (X, τ) be a topological space. Prove that a subset A of X is open if and only if it is a neighborhood of each of its points.

   ii. Let $X = \{a, b, c\}$ and $\tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}\}$ be a topology on X. Find $N(a), N(b)$ and $N(c)$. (That is, neighborhoods of 'a', 'b' and 'c').

   (d) Let (Y, τ_Y) be a subspace of a topological space (X, τ). Prove that $A \subseteq Y$ is closed in (Y, τ_Y) if and only if $A = F \cap Y$ for some closed subset F of X.

2. (a) Define the following terms in a topological space (X, τ) :

   i. Base;

   ii. Subbase;

   iii. Disconnected Set.

(b) Let X be a non - empty set and let $\mathbb{B}$ be a collection of subsets of X such a way that

i. $X = \bigcup_{B_i \in \mathbb{B}} B_i$,

ii. $\forall B_1, B_2 \in \mathbb{B}$ and $\forall x \in B_1 \cap B_2, \exists B \in \mathbb{B}$ such that $x \in B \subseteq B_1 \cap$

Prove that there exist a unique topology τ for X such that $\mathbb{B}$ is a ba

(c) Prove that a topological space (X, τ) is disconnected if and only if th a non - empty proper subset of X which is both open and closed.

(d) Let (X, τ) be a topological space. Prove that X is disconnected if an there are non - empty subsets A, B of X such that $X = A \cup B$ and $\bar{A} \cap B = A \cap \bar{B} = \phi$.

3. Explain what is meant by the statement that A is a compact subset of a to space (X, τ).

(a) Let (X, τ) be a topological space and let (Y, τ_Y) be its subspace and le Prove that A is compact in (Y, τ_Y) if and only if A is compact in $(X,$

(b) Prove that continuous image of a compact subset in a topological compact.

(c) Prove that continuous image of a sequentially compact set is sequentia pact.

(d) Let A and B be two compact subsets of a topological space (X, τ). Pr $(A \cup B)$ is compact.

4. (a) What is meant by a function f from a topological space (X, τ_1) to a to space (Y, τ_2) is continuous at a point $x_0 \in X$?.

i. Let (X, τ_1) and (Y, τ_2) be two topological spaces and let $f : X -$ function. Prove that f is continuous on X if and only if $f^{-1}(F)$ in (X, τ_1) for each closed set F in (Y, τ_2).

ii. Suppose that (X, τ_1) and (Y, τ_2) are topological spaces and $f : X$ a function and $\mathbb{B}$ is any basis for τ_2. Prove that f is continuous and only if for each $B \in \mathbb{B}, f^{-1}(B)$ is an open set in X.

(b) Define Frechet Space (T_1 - Space) and Hausdorff Space (T_2 - Space).

i. Prove that every T_2 - Space is a T_1 - Space. Is the converse true? Justify your answer.

ii. Prove that a topological space X is a T_1 - Space if and only if every singleton subset of X is closed.