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CAUSAL HEAT FLOW IN BIANCHI TYPE-V UNIVERSE

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In this paper we investigate the role of causal heat transport in a spatially homogeneous, locally-rotationally symmetric Bianchi type-V cosmological model. In particular, the causal temperature profile of the cosmological fluid is obtained within the framework of extended irreversible thermodynamics. We demonstrate that relaxational effects can alter the temperature profile when the cosmological fluid is out of hydrostatic equilibrium.

Keywords: Cosmology; Homogeneity; Thermodynamics.

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1. Introduction

While the standard Big Bang cosmological model has accounted for observations of homogeneity and isotropy of the Universe on large scales there are still open questions regarding the identification of the dark energy components making up the cosmic fluid.[1] This has led to the pursuit of more general models in which the geometry and the matter content have drastically changed when compared to the standard FRW cosmologies. To date the ΛCDM concordance model has proved to be highly successful in accounting for all current observations ranging from supernovae Ia, CMBR anisotropies,weak lensing, baryon oscillations through to large-scale structure formation.[2] There are various alternative cosmological models ranging from inhomogeneous cosmologies with dissipative fluxes,[3,4] singularity-free models,[5,6] emergent Universe models[7,8] and the spatially homogeneous Bianchi models.[9,10] It is claimed that these models can account for many of the mechanisms leading to the current state of the Universe such as inflation, particle production and anisotropy in the infant Universe. The homogeneous and isotropic FRW cosmological models

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are particular cases of the Bianchi I, V and IX universes, depending on the constant curvature of the physical three-space, $t = $ constant. In particular, the Bianchi V universe is a simple generalisation of the negative curvature FRW models.[9, 10] These alternative models can represent particular epochs during the evolution of our Universe. As pointed out by Ellis[11] the anisotropic Bianchi-type cosmologies are worthy of attention even if current observations indicate that our Universe is FLWR-like in nature. The observed isotropy of the present Universe does not rule out the possibility of dominant anisotropic effects in the early Universe.

The role of dissipation in inhomogeneous cosmological models have been widely studied. Romano and Pavon studied the evolution of the Bianchi type-I model with viscous dissipation within the framework of extended irreversible thermodynamics.[12] They were able to show that the Bianchi type-I cosmological model does not asymptotically evolve into the Friedmann or de-Sitter phase. This is mainly due to relaxational effects within the cosmological fluid. Romano and Pavon utilised both the truncated and full causal thermodynamic theory to study the evolution of the Bianchi type-III cosmological models. Their results show that there is rapid dissipation of the initial anisotropies leading to stable de Sitter solutions while the Friedmann ones are unstable.[13] Causal heat transport in an inhomogeneous cosmological model was investigated by Triginer and Pavon.[14] By imposing a barotropic equation of state and by employing a heat transport equation of Maxwell-Cattaneo form they were able obtain more general behaviour of the scale factor and entropy production for various spherically symmetric, inhomogeneous cosmological models. Singh and Beesham[16] investigated the effect of heat flow in a LRS Bianchi type-V universe with constant deceleration parameter. They calculated the temperature distribution for the cosmological fluid by employing the Eckart transport equation for the heat flow.

In this paper we revisit the model investigated by Singh and Beesham with the view of highlighting the relaxational effects on the temperature distribution. To this end we employ a causal heat transport equation of Maxwell-Cattaneo form. By assuming that the relaxation time is inversely proportional to the inverse of the absolute value of the expansion of the cosmic fluid we are able to integrate the truncated heat transport equation to obtain the temperature profile. Our results show distinct differences between the causal and noncausal temperatures throughout the cosmic fluid.

2. LRS Bianchi type-V cosmology

The line element for locally-rotationally symmetric Bianchi type-V cosmological model is given by[16]

$$ds^2 = -dt^2 + A^2 dx^2 + e^{2x} B^2 \left(dy^2 + dz^2 \right), \tag{1}$$

where $A = A(t)$ and $B = B(t)$ are metric functions yet to be determined. The matter distribution for the cosmological fluid interior is represented by the energy

momentum tensor of an imperfect fluid

$$T_{ab} = (\rho + p)u_a u_b + p g_{ab} + Q_a u_b + Q_b u_a, \tag{2}$$

where ρ is the energy density, p is the pressure and $Q = (Q^a Q_a)^{\frac{1}{2}}$ is the magnitude of the heat flux. The fluid four–velocity **u** is comoving and is given by

$$u^a = \delta_0^a. \tag{3}$$

The heat flow vector takes the form

$$Q^a = (0, Q^1, 0, 0), \tag{4}$$

since $Q^a u_a = 0$ and the heat is assumed to flow in the radial direction. The fluid collapse rate $\Theta = u^a_{;a}$ of the stellar model is given by

$$\Theta = \frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}. \tag{5}$$

The Einstein field equations reduce to

$$\rho = 2\frac{\dot{A}}{A}\frac{\dot{B}}{B} + \frac{\dot{B}^2}{B^2} - \frac{3}{A^2}, \tag{6}$$

$$p = \frac{1}{A^2} - \frac{\dot{B}^2}{B^2} - 2\frac{\ddot{B}}{B}, \tag{7}$$

$$p = \frac{1}{A^2} - \frac{\dot{A}}{A}\frac{\dot{B}}{B} - \frac{\ddot{A}}{A} - \frac{\ddot{B}}{B}, \tag{8}$$

$$Q_1 = 2\left(\frac{\dot{B}}{B} - \frac{\dot{A}}{A}\right), \tag{9}$$

for the line element (1). The generalized mean Hubble parameter H is given by

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right), \tag{10}$$

where $a = (AB^2)^{1/3}$ is the average scale factor. The dot denotes a derivative with respect to cosmic time t.

3. Evolution of Hubble parameter

Observations of the CMB and SNe Ia data point to an accelerating universe ($q < 0$) where q is the deceleration parameter. Following Singh et al[10] we assume that the Hubble parameter is related to the average scale factor by

$$H = la^{-n} = l(AB^2)^{-n/3}, \tag{11}$$

where $l(> 0)$ and $n(\geq 0)$ are constants, and

$$n = q + 1, \tag{12}$$

where H is defined as in eq. (10) and q the deceleration parameter defined by

$$q = -\frac{\ddot{a}a}{\dot{a}^2}. \tag{13}$$

Using eqs. (11) and (12), the solution of eq. (13) gives the law of variation of average scale factor of the form

$$a = (nlt)^{1/n}, \tag{14}$$

for $n \neq 0$ and

$$a = c\exp[lt], \tag{15}$$

for $n = 0$, where c is the constant of integration. Here, in eq. (14), we have assumed that for $t = 0$ the value $a = 0$ so that the constant of integration vanishes.

Now, from eqs. (7) and (8), we get

$$\frac{\ddot{B}}{B} - \frac{\ddot{A}}{A} + \frac{\dot{B}^2}{B^2} - \frac{\dot{A}}{A}\frac{\dot{B}}{B} = 0. \tag{16}$$

Integrating eq. (16) and utilizing $a = (AB^2)^{1/3}$, the metric functions A and B can be expressed as quadratures

$$A(t) = (d_1)^{-2/3}a\exp\left[-\frac{2k_1}{3}\int a^{-3}dt\right], \tag{17}$$

$$B(t) = (d_1)^{1/3}a\exp\left[\frac{k_1}{3}\int a^{-3}dt\right], \tag{18}$$

where k_1 and d_1 are the constants of integration.

The case $n \neq 0$ was considered by Singh and Beesham.[16] Our aim is to investigate relaxational effects when the cosmic fluid leaves hydrostatic equilibrium. To this end we consider the case $n = 0$ which is equivalent to $q = -1$ which corresponds to inflation. Substituting eq. (15) into eqs. (17) and (18), the solution of the metric functions is given by

$$A(t) = (d_1)^{-2/3}c\exp\left[lt + \frac{2k_1}{3lc^3}\exp(-3lt)\right], \tag{19}$$

$$B(t) = (d_1)^{1/3}c\exp\left[lt - \frac{k_1}{3lc^3}\exp(-3lt)\right]. \tag{20}$$

The heat flow is given by

$$Q_1 = \frac{2k_1}{c^3}\exp(-3lt). \tag{21}$$

The energy density and pressure are respectively given by

$$\rho = 3l^2 - \frac{1}{3}\frac{k_1^2}{c^6}\exp(-6lt)$$
$$-3(d_1)^{4/3}c^{-2}\exp\left[-2\left(lt + \frac{2k_1}{3lc^3}\exp[-3lt]\right)\right], \tag{22}$$

$$p = -3l^2 - \frac{1}{3}\frac{k_1^2}{c^6}\exp(-6lt)$$
$$+(d_1)^{4/3}c^{-2}\exp\left[-2\left(lt + \frac{2k_1}{3lc^3}\exp[-3lt]\right)\right]. \tag{23}$$

As pointed out by Singh and Beesham, the Universe as described by this model starts evolving with constant kinematical and thermodynamical parameters and maintains a constant expansion rate. At late times this model mimicks an inflationary-like behaviour with an equation of state $p = -\rho$. Inflation driven by heat flux was demonstrated by Maartens *et al*[3] in which they showed that the heat flux serves to 'balance' the decrease in energy density while the pressure of the cosmic fluid steadily decreases. In order to determine the deviation of the cosmic fluid from hydrostatic equilibrium we calculate the covariant dimensionless ratio

$$\frac{|Q|}{\rho} = 2\frac{\sqrt{\frac{(d_1)^{4/3}k_1^2\exp[-\frac{\exp[-3ltk_1]-8lt]}{3c^3l}]}{c^8}}}{-\frac{3(d_1)^{3/2}\exp[\frac{-\exp[-3ltk_1]-2lt]}{3c^3l}]}{c^2} - \frac{k_1^2\exp[-6lt]}{3c^6} + 3l^2}, \tag{24}$$

which for late times decreases rapidly indicating that $|Q|$ decreases less rapidly than ρ during this epoch. Herrera *et al*[18] have shown that a certain parameter α defined by

$$\alpha = \frac{1}{(\rho + p)}\left(\frac{\zeta}{2\tau_\zeta} + \frac{\kappa T}{\tau_\kappa} + \frac{2\eta}{3\tau_\eta}\right),$$

where ζ, κ and η are the transport coefficients of bulk viscosity, heat conduction and shear viscosity, respectively and τ_i^s are the corresponding relaxation times, is a measure of the strength of expansion during the inflationary phase. Larger values of α lead to stronger expansion. Furthermore, more efficient models of inflation can be constructed by including bulk viscosity, heat conduction and shear viscosity thus strengthening the case for inhomogeneous cosmological models.

4. Causal Thermodynamics

In order to study the influence of relaxational effects when the cosmic fluid departs from hydrostatic equilibrium we employ a causal heat transport equation of Maxwell-Cattaneo form given by[19–22]

$$\tau h_a{}^b\dot{Q}_b + Q_a = -\kappa\left(h_a{}^b\nabla_b T + T\dot{u}_a\right) \tag{25}$$

where $h_{ab} = g_{ab} + u_a u_b$ projects into the comoving rest space, T is the local equilibrium temperature, κ (≥ 0) is the thermal conductivity, and τ (≥ 0) is the relaxational time-scale which gives rise to the causal and stable behaviour of the theory.

The noncausal Fourier heat transport equation is obtained by setting $\tau = 0$ in (25). For the metric (1), equation (25) becomes

$$\tau(QA)^{\cdot} + QA = -\frac{\kappa(T)'}{A} \tag{26}$$

where T' represents the temperature gradient. Note that on setting $\tau = 0$, we regain the Eckart heat transport equation

$$QA = -\frac{\kappa(T)'}{A} \tag{27}$$

which was utilised by Singh and Beesham to obtain noncausal temperature profiles. Following Triginer and Pavon,[14] we assume the thermal conductivity is that for a radiation fluid interacting with matter

$$\kappa = cT^3\sigma, \tag{28}$$

where $c > 0$ is a constant and σ is the mean collision time. The mean collision time is related to the particle number density n via

$$\sigma = \alpha n^{-1/3}, \tag{29}$$

where $\alpha > 0$ is an arbitrary constant. The particle conservation equation

$$\frac{dn}{ds} + n\Theta = 0, \tag{30}$$

which yields $n \propto (AB^2)^{-1}$. We can finally write

$$\sigma = \alpha(AB^2)^{1/3}, \tag{31}$$

where $\alpha > 0$ is another constant. Since Θ^{-1} is the only natural time scale of the cosmological fluid, we define the relaxation time as follows

$$\tau = \beta|\Theta^{-1}| = \beta\left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B}\right)^{-1} \tag{32}$$

where $\beta(\geq 0)$ can be viewed as a causality 'switch'. By setting $\beta = 0$ we regain the noncausal Eckart transport equation. The mean collision time is related to the relaxation time via (29), (30) and (32). We can write

$$\sigma = \sigma_0 e^{-\int \frac{\beta}{\tau} dt} \tag{33}$$

where $\sigma_0 > 0$ is a constant. Utilising (19) and (20) in (32) we obtain

$$\tau = \frac{\beta}{3l} \tag{34}$$

which corresponds to constant relaxation time. From (33), we can immediately write

$$\sigma = \sigma_0 e^{-3lt} \tag{35}$$

. We can conclude that the assumption made in (32) holds to good approximation in the early evolution of the cosmological fluid when temperatures are sufficiently high.[15] The causal heat transport equation (26) becomes

$$\beta \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \right)^{-1} (QA)\dot{} + QA = -c_0 T^3 \frac{(AB^2)^{1/3}}{A} T', \qquad (36)$$

which easily integrates to

$$T^4 = -\frac{4}{c_0} \left(\frac{A}{B} \right)^{2/3} \left[\beta \left(\frac{\dot{A}}{A} + 2\frac{\dot{B}}{B} \right)^{-1} (QA)\dot{} + QA \right] x + \mathcal{F}(t), \qquad (37)$$

where

$$Q = \frac{2}{A^2} \left[\frac{\dot{B}}{B} - \frac{\dot{A}}{A} \right] \qquad (38)$$

The above equation is readily solved to give us the noncausal temperature profiles for the two cases that we have investigated thus far. For $n = 0$ which corresponds to the case of constant deceleration parameter $q = -1$ yields

$$T^4 = \frac{-24k_1}{c_0 c^4} \exp(-4lt) \left[\beta \left(\frac{k_1}{c^3} \exp(-3lt) - 2 \right) + 1 \right] x + \mathcal{F}_2(t) \qquad (39)$$

where $\mathcal{F}_2(t)$ is a function of integration. Note that (39) does not guarantee $T > 0$. Requiring $T > 0$ on physical grounds will constrain the free parameters appearing in (39). The noncausal temperature is obtained by setting $\beta = 0$ in (39). This would imply (from (32)) that thermal equilibrium is achieved instantaneously which is one of the pathologies of the Eckart theory. It has been pointed out that entropy of the Universe behaves like a an ordinary system and tends to a maximum value of the order of H^{-2} as $a \to \infty$.[17] The rate of entropy production is given by

$$S^a_{;a} = \frac{Q^a Q_a}{T^2} = \frac{4(d_1)^{4/3} k_1^2 \exp[-\frac{4k_1 \exp[-3lt]}{3c^3 l} - 8lt]}{c^8 \sqrt{-24 \frac{c^4 k_1 x \exp[-4lt](1+(-2+k_1 \exp[-3lt]/c^3)\beta)}{c_0}} + \mathcal{F}_2(t)},$$

which vanishes as $t \to \infty$ for an appropriate choice of $\mathcal{F}$. Let us consider the temperature profile for the case $n = 0$. Figures 1 and 2 show the evolution of the causal and noncausal temperature profiles respectively. In order to generate these plots we chose the following parameters: $d_1 = 1, k_1 = 10000, c = 0.001, l = 1, c_0 = -1$. Figure 1 corresponds to the case $\beta = 0$ giving the noncausal temperature. Figure 2 corresponds to the case $\beta = 1000$ representing the causal temperature profile. It is evident that the causal temperature is everywhere greater than its noncausal counterpart. In the infinite past both the causal and noncausal temperatures are at a maximum and decrease as the fluid evolves with time. The drop-off in the temperature is greater in the causal case than the noncausal case indicating that cooling is enhanced by relaxational effects.

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5. Concluding remarks

We have successfully obtained the causal temperature profile for an LRS Bianchi type-V cosmological fluid with constant deceleration parameter. Our results generalise the thermodynamical results obtained by Singh and Beesham.[16] Our investigation show that relaxational effects within the cosmic fluid leads to a higher temperature . We also found that the rate of entropy production decreases as the Universe evolves in time, tending to zero for late times. It would be interesting to investigate the evolution of the temperature profile for the LRS Bianchi type-V universe with dissipation by employing a full causal heat transport equation. Work in this direction has been initiated.

Acknowledgments

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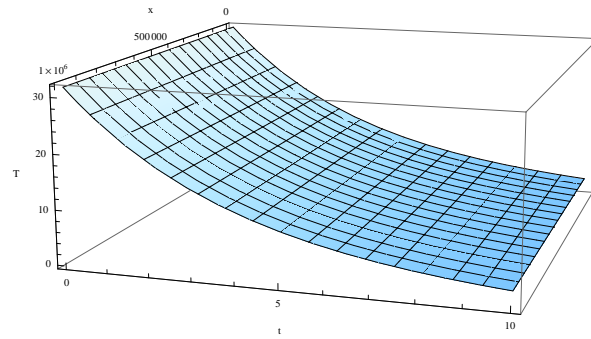22. R. Maartens, *Causal Thermodynamics in Relativity* arXiv:astro-ph/9609119.

Fig. 1.   Noncausal temperature as a function of the radial x and temporal t coordinates
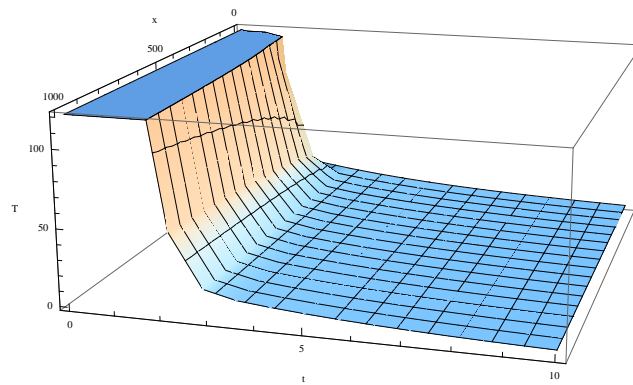

Fig. 2.   Causal temperature as a function of the radial x and temporal t coordinates