Charged anisotropic matter with linear equation of state

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Abstract

We consider the general situation of a compact relativistic body with anisotropic pressures in the presence of the electromagnetic field. The equation of state for the matter distribution is linear and may be applied to strange stars with quark matter. Three classes of new exact solutions are found to the Einstein-Maxwell system. This is achieved by specifying a particular form for one of the gravitational potentials and the electric field intensity. We can regain anisotropic and isotropic models from our general class of solution. A physical analysis indicates that the charged solutions describe realistic compact spheres with anisotropic matter distribution. The equation of state is consistent with dark energy stars and charged quark matter distributions. The masses and central densities correspond to realistic stellar objects in the general case when anisotropy and charge are present.

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1 Introduction

Since the pioneering paper by Bowers and Liang [1] there have been extensive investigations in the study of anisotropic relativistic matter distributions in general relativity to include the effects of spacetime curvature. The anisotropic interior spacetime matches to the Schwarzschild exterior model. The early work of Ruderman [2] showed that nuclear matter may be anisotropic in density ranges of 10^{15} gcm^{-3} where nuclear interactions need to be treated relativistically. Note that conventional celestial bodies are not composed purely of perfect fluids so that radial pressures are different from tangential pressures. Anisotropy can be introduced by the existence of a solid stellar core or by the presence of a type 3A superfluid as indicated by Kippenhahn and Weigert [3]. Different kinds of phase transitions (Sokolov [4]) or pion condensation (Sawyer [5]) can generate anisotropy. Binney and Tremaine [6] have considered anisotropies in spherical galaxies in the context of Newtonian gravitational theory. Herrera and Santos [7] studied the effects of slow rotation in stars and Letelier [8] analysed the mixture of two gases, such as ionized hydrogen and electrons, in a framework of a relativistic anisotropic fluid. Weber [9] showed that strong magnetic fields serve as a vehicle for generating anisotropic pressures inside a compact sphere. Some recent anisotropic models for compact self-gravitating objects with strange matter include the results of Lobo [10] and Sharma and Maharaj [11] with a barotropic equation of state. Therefore the study of anisotropic fluid spheres in static spherically symmetric spacetimes is important in relativistic astrophysics.

In recent years there have been several investigations of the Einstein-Maxwell system of equations for static spherically symmetric gravitational fields usually with isotropic pressures to include the effects of the electromagnetic field. The interior spacetime must match at the boundary to the Reissner-Nordstrom exterior model. The models generated can be used to describe charged relativistic bodies in strong gravitational fields such as neutron stars. Many exact solutions have been given by Ivanov [12] and Thirukkanesh and Maharaj [13] which satisfy the conditions for a physically acceptable charged relativistic sphere. Charged spheroidal stars have been studied extensively by Komathiraj and Maharaj [14], Sharma *et al* [15], Patel and Koppor [16], Tikekar and Singh [17] and Gupta and Kumar [18]. These charged spheroidal models contain uncharged neutron stars in the relevant limit and are consequently relevant in the description of dense astrophysical objects. We point out the particular detailed studies of Sharma *et al* [19] in cold compact objects, Sharma and Mukherjee [20] analysis of strange matter and binary pulsars, and Sharma and Mukherjee [21] analysis of quark-diquark mixtures in equilibrium in the presence of the electromagnetic field. Charged relativistic matter is also relevant in modeling core-envelope stellar system

as shown in the treatments of Thomas *et al* [22], Tikekar and Thomas [23] and Paul and Tikekar [24] in which the stellar core is an isotropic fluid surrounded by a layer of anisotropic fluid. Consequently the study of charged fluid spheres in static spherically symmetric spacetimes is of significance in relativistic astrophysics.

From the above motivation it is clear that both anisotropy and the electromagnetic field are important in astrophysical processes. However previous treatments have largely considered either anisotropy or electromagnetic field separately. The intention of this paper is to provide a general framework that admits the possibility of tangential pressures with a nonvanishing electric field intensity. We believe that this approach will allow for a richer family of solutions to the Einstein-Maxwell field equations and possibly provide a deeper insight into the behaviour of the gravitational field. On physical grounds we impose a barotropic equation of state which is linear, that relates the radial pressure to the energy density and allows for the existence of strange matter. Our general model will contain strange matter solutions found previously. In this regard we mention the following recent works on strange stars. Mak and Harko [25] and Komathiraj and Maharaj [26] found analytical models in the MIT bag model (Witten [27]) with a strange matter equation of state in the presence of an electromagnetic field. Sharma and Maharaj [11] generated a class of exact solutions which can be applied to strange stars with quark matter for neutral anisotropic matter. Lobo [10] found stable dark energy stars which generalise the gravastar model governed by a dark energy equation of state.

The objective of this treatment is to generate exact solutions to the Einstein-Maxwell system, with linear equation of state, that may be utilised to describe a charged anisotropic relativistic body. In Section 2, we express the Einstein-Maxwell system as a new system of differential equations using a coordinate transformation, and then write the system in another form which is easier to analyse. Three classes of new exact solutions to the Einstein-Maxwell system are found in Section 3 in terms of simple elementary functions. We show that particular uncharged anisotropic strange stars found in the past are contained in our general family of solutions. In Section 4, we show that the solutions are physically admissible and plot the matter variables for particular parameter values. We generate values for the mass and central density in Section 5 for charged and uncharged matter. This analysis extends the treatment of Sharma and Maharaj [11] to include charge, and confirms that the exact solutions found are physically reasonable. Some concluding remarks are made in Section 6.

2   The field equations

Our intention is to model the interior of a dense star. On physical grounds it is necessary for the gravitational field to be static and spherically symmetric. Consequently, we assume that the interior of a spherically symmetric star is described by the line element

$$ds^2 = -e^{2\nu(r)}dt^2 + e^{2\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2) \tag{1}$$

in Schwarzschild coordinates $(x^a) = (t, r, \theta, \phi)$. We take the energy momentum tensor for an anisotropic charged imperfect fluid sphere to be of the form

$$T_{ij} = \text{diag}(-\rho - \frac{1}{2}E^2, p_r - \frac{1}{2}E^2, p_t + \frac{1}{2}E^2, p_t + \frac{1}{2}E^2), \tag{2}$$

where ρ is the energy density, p_r is the radial pressure, p_t is the tangential pressure and E is the electric field intensity. These quantities are measured relative to the comoving fluid velocity $u^i = e^{-\nu}\delta_0^i$. For the line element (1) and matter distribution (2) the Einstein field equations can be expressed as

$$\frac{1}{r^2}\left[r(1 - e^{-2\lambda})\right]' = \rho + \frac{1}{2}E^2, \tag{3}$$

$$-\frac{1}{r^2}\left(1 - e^{-2\lambda}\right) + \frac{2\nu'}{r}e^{-2\lambda} = p_r - \frac{1}{2}E^2, \tag{4}$$

$$e^{-2\lambda}\left(\nu'' + \nu'^2 + \frac{\nu'}{r} - \nu'\lambda' - \frac{\lambda'}{r}\right) = p_t + \frac{1}{2}E^2, \tag{5}$$

$$\sigma = \frac{1}{r^2}e^{-\lambda}(r^2 E)', \tag{6}$$

where primes denote differentiation with respect to r and σ is the proper charge density. In the field equations (3)-(6), we are using units where the coupling constant $\frac{8\pi G}{c^4} = 1$ and the speed of light $c = 1$. The system of equations (3)-(6) governs the behaviour of the gravitational field for an anisotropic charged imperfect fluid. Note that the system (3)-(6) becomes

$$\frac{1}{r^2}\left[r(1 - e^{-2\lambda})\right]' = \rho, \tag{7}$$

$$-\frac{1}{r^2}\left(1 - e^{-2\lambda}\right) + \frac{2\nu'}{r}e^{-2\lambda} = p, \tag{8}$$

$$e^{-2\lambda}\left(\nu'' + \nu'^2 + \frac{\nu'}{r} - \nu'\lambda' - \frac{\lambda'}{r}\right) = p, \tag{9}$$

for matter distributions with isotropic pressures $(p_r = p_t)$ in the absence of charge $(E = 0)$.

The mass contained within a radius r of the sphere is defined as

$$m(r) = \frac{1}{2}\int_0^r \omega^2 \rho(\omega)d\omega. \tag{10}$$

4

A different, but equivalent, form of the field equations is obtained if we introduce a new independent variable x, and define functions y and Z, as follows

$$x = Cr^2, \quad Z(x) = e^{-2\lambda(r)} \text{ and } A^2 y^2(x) = e^{2\nu(r)}, \tag{11}$$

which was first suggested by Durgapal and Bannerji [28]. Then the line element (1) becomes

$$ds^2 = -A^2 y^2 dt^2 + \frac{1}{4CxZ} dx^2 + \frac{x}{C}(d\theta^2 + \sin^2\theta d\phi^2). \tag{12}$$

In (11) and (12), the quantities A and C are arbitrary constants. Under the transformation (11), the system (3)-(6) becomes

$$\frac{1-Z}{x} - 2\dot{Z} = \frac{\rho}{C} + \frac{E^2}{2C}, \tag{13}$$

$$4Z\frac{\dot{y}}{y} + \frac{Z-1}{x} = \frac{p_r}{C} - \frac{E^2}{2C}, \tag{14}$$

$$4xZ\frac{\ddot{y}}{y} + (4Z + 2x\dot{Z})\frac{\dot{y}}{y} + \dot{Z} = \frac{p_t}{C} + \frac{E^2}{2C}, \tag{15}$$

$$\frac{\sigma^2}{C} = \frac{4Z}{x}\left(x\dot{E} + E\right)^2, \tag{16}$$

where dots denote differentiation with respect to the variable x. The mass function (10) becomes

$$m(x) = \frac{1}{4C^{3/2}}\int_0^x \sqrt{w}\rho(w)dw, \tag{17}$$

in terms of the new variables in (11).

For a physically realistic relativistic star we expect that the matter distribution should satisfy a barotropic equation of state $p_r = p_r(\rho)$. For our purposes we assume the linear equation of state

$$p_r = \alpha\rho - \beta, \tag{18}$$

where α and β are constants. Then it is possible to write the system (13)-(16) in the simpler form

$$\frac{\rho}{C} = \frac{1-Z}{x} - 2\dot{Z} - \frac{E^2}{2C}, \tag{19}$$

$$p_r = \alpha\rho - \beta, \tag{20}$$

$$p_t = p_r + \Delta, \tag{21}$$

$$\Delta = 4CxZ\frac{\ddot{y}}{y} + 2C\left[x\dot{Z} + \frac{4Z}{(1+\alpha)}\right]\frac{\dot{y}}{y}$$
$$+ \frac{(1+5\alpha)}{(1+\alpha)}C\dot{Z} - \frac{C(1-Z)}{x} + \frac{2\beta}{(1+\alpha)}, \tag{22}$$

$$\frac{E^2}{2C} = \frac{1-Z}{x} - \frac{1}{(1+\alpha)}\left[2\alpha\dot{Z} + 4Z\frac{\dot{y}}{y} + \frac{\beta}{C}\right], \tag{23}$$

$$\frac{\sigma^2}{C} = 4\frac{Z}{x}(x\dot{E} + E)^2, \tag{24}$$

where the quantity $\Delta = p_t - p_r$ is the measure of anisotropy in this model. In the system (19)-(24), there are eight independent variables $(\rho, p_r, p_t, \Delta, E, \sigma, y, Z)$ and only six independent equations. This suggests that it is possible to specify two of the quantities involved in the integration process. The resultant system will remain highly nonlinear but it may be possible to generate exact solutions.

3   Generating exact models

We must make physically reasonable choices for any two of the independent variables and then solve the system (19)-(24) to generate exact models. In this paper, we choose forms for the gravitational potential Z and electric field intensity E. We make the specific choices

$$Z = \frac{1 + (a - b)x}{1 + ax}, \tag{25}$$

$$\frac{E^2}{C} = \frac{k(3 + ax)}{(1 + ax)^2}, \tag{26}$$

where a, b and k are real constants. The gravitational potential Z is regular at the origin and well behaved in the stellar interior for a wide range of values for the parameters a and b. The electric field intensity is continuous, bounded and a decreasing function from the origin to the boundary of the sphere. Therefore the forms chosen in (25)-(26) are physically reasonable. On substituting (25) and (26) in (23) we obtain

$$\begin{aligned}
\frac{\dot{y}}{y} &= \frac{(1 + \alpha)b}{4\left[1 + (a - b)x\right]} + \frac{\alpha b}{2(1 + ax)\left[1 + (a - b)x\right]} \\
&\quad - \frac{\beta(1 + ax)}{4C\left[1 + (a - b)x\right]} - \frac{(1 + \alpha)k(3 + ax)}{8(1 + ax)\left[1 + (a - b)x\right]},
\end{aligned} \tag{27}$$

which is a linear equation in the gravitational potential y. For the integration of equation (27) it is convenient to consider three cases: $b = 0, a = b$ and $a \neq b$.

3.1   The case $b = 0$

When $b = 0$, (27) becomes

$$\frac{\dot{y}}{y} = -\frac{\beta}{4C} - \frac{(1 + \alpha)k(3 + ax)}{8(1 + ax)^2} \tag{28}$$

with solution

$$y = D(1 + ax)^{\frac{-k(1+\alpha)}{a}} \exp\left[\frac{2k(1 + \alpha)}{a(1 + ax)} - \frac{\beta x}{4C}\right], \tag{29}$$

where D is the constant of integration. We observe that $\rho = -\frac{E^2}{2}$ for this case which we do not consider further to avoid negative energy densities.

3.2 The case $a = b$

When $a = b$, (27) becomes

$$\frac{\dot{y}}{y} = \frac{(1+\alpha)a}{4} + \frac{\alpha a}{2(1+ax)} - \frac{\beta(1+ax)}{4C} - \frac{(1+\alpha)k(3+ax)}{8(1+ax)}. \tag{30}$$

On integrating (30) we get

$$y = D(1+ax)^{\frac{2a\alpha-(1+\alpha)k}{4a}} \exp[F(x)], \tag{31}$$

where

$$F(x) = \frac{x}{8C}[-kC(1+\alpha) - 2\beta + a(2C(1+\alpha) - \beta x)]$$

and D is the constant of integration. Then we can generate an exact model for the system (19)-(24) as follows

$$e^{2\lambda} = 1 + ax, \tag{32}$$

$$e^{2\nu} = A^2 D^2 (1+ax)^{\frac{2a\alpha-k(1+\alpha)}{2a}} \exp[2F(x)], \tag{33}$$

$$\frac{\rho}{C} = \frac{(2a-k)}{2}\frac{(3+ax)}{(1+ax)^2}, \tag{34}$$

$$p_r = \alpha\rho - \beta, \tag{35}$$

$$p_t = p_r + \Delta, \tag{36}$$

$$\begin{aligned}
\Delta = \ & \frac{1}{16C(1+ax)^3}\Big\{C^2\big[k^2(1+\alpha)^2 x(3+ax)^2 \\
& + 4a^2 x(3 - 8\alpha + 9\alpha^2 + a^2(1+\alpha)^2 x^2 + 2ax(2 + 3\alpha + 3\alpha^2)) \\
& - 4k(12 + a^3(1+\alpha)^2 x^3 + a^2 x^2(7 + 9\alpha + 6\alpha^2) + ax(12 + 5\alpha + 9\alpha^2))\big] \\
& - 4Cx(1+ax)^2[(1+\alpha)(2a^2 x - 3k) - a\beta(k(1+\alpha) - 6\alpha - 4)] \\
& + 4\beta^2 x(1+ax)^4\Big\},
\end{aligned} \tag{37}$$

$$\frac{E^2}{C} = \frac{k(3+ax)}{(1+ax)^2}, \tag{38}$$

in terms of elementary functions.

The solution (32)-(38) may be used to model a charged anisotropic star with a linear equation of state. In this case the mass function is

$$m(x) = \frac{(2a-k)x^{3/2}}{4C^{3/2}(1+ax)}, \tag{39}$$

which is similar to forms used in other investigations. The gravitational potentials and matter variables are continuous and well behaved in the stellar interior. Note that when $k = 0$ the model (32)-(38) reduces to a solution for uncharged anisotropic stars. Equation (37) yields

$$\begin{aligned}
\Delta = \ & \frac{1}{4C(1+ax)^3}\Big\{C^2 a^2 x\big[3 - 8\alpha + 9\alpha^2 + a^2(1+\alpha)^2 x^2 + 2ax(2 + 3\alpha + 3\alpha^2)\big] \\
& - 2Cx(1+ax)^2[(1+\alpha)a^2 x + a\beta(3\alpha + 2)] + \beta^2 x(1+ax)^4\Big\}
\end{aligned} \tag{40}$$

when $k = 0$ so that the model is necessarily anisotropic with $\Delta \neq 0$ in general even in the simpler case of uncharged matter. Some treatments of the physical properties of anisotropic spheres in general relativity include the investigations of Dev and Gleiser [29, 30], Mak and Harko [31, 32], Chaisi and Maharaj [33, 34] and Maharaj and Chaisi [35] with $\Delta \neq 0$.

3.3 The case $a \neq b$

On integrating (27) we get

$$y = D(1 + ax)^m [1 + (a - b)x]^n \exp\left[\frac{-a\beta x}{4C(a - b)}\right] \tag{41}$$

where D is the constant of integration, and m and n are given by

$$
\begin{aligned}
m &= \frac{2\alpha b - (1 + \alpha)k}{4b}, \\
n &= \frac{1}{8bC(a - b)^2} \left[2a^2C(k(1 + \alpha) - 2\alpha b) - abC(5k(1 + \alpha) - 2b(1 + 5\alpha)) \right. \\
&\quad \left. + b^2(3kC(1 + \alpha) - 2bC(1 + 3\alpha) + 2\beta)\right].
\end{aligned}
$$

Then we can generate an exact model for the system (19)-(24) in the form

$$e^{2\lambda} = \frac{1 + ax}{1 + (a - b)x}, \tag{42}$$

$$e^{2\nu} = A^2 D^2 (1 + ax)^{2m} [1 + (a - b)x]^{2n} \exp\left[\frac{-a\beta x}{2C(a - b)}\right], \tag{43}$$

$$\frac{\rho}{C} = \frac{(2b - k)}{2} \frac{(3 + ax)}{(1 + ax)^2}, \tag{44}$$

$$p_r = \alpha\rho - \beta, \tag{45}$$

$$p_t = p_r + \Delta, \tag{46}$$

$$
\begin{aligned}
\Delta &= \frac{-bC}{(1 + ax)} - \frac{bC(1 + 5\alpha)}{(1 + \alpha)(1 + ax)^2} + \frac{2\beta}{(1 + \alpha)} \\
&\quad + \frac{Cx[1 + (a - b)x]}{(1 + ax)} \left[4\left(\frac{a^2 m(m - 1)}{(1 + ax)^2} + \frac{2a(a - b)mn}{(1 + ax)[1 + (a - b)x]}\right.\right. \\
&\quad \left. + \frac{(a - b)^2 n(n - 1)}{[1 + (a - b)x]^2}\right) - \frac{2a\beta(a(m + n)[1 + (a - b)x] - bn)}{(a - b)C(1 + ax)[1 + (a - b)x]} + \frac{a^2\beta^2}{4C^2(a - b)^2}\right] \\
&\quad - \frac{4[1 + ax(2 + (a - b)x)] - b(5 + \alpha)x}{2(a - b)(1 + \alpha)(1 + ax)^3[1 + (a - b)x]} \times \\
&\quad \left[-4b^2 Cn + a^3 x(-4C(m + n) + \beta x) + a^2(4C(m + n)(2bx - 1) + \beta(2 - bx)x)\right. \\
&\quad \left. + a(-4b^2 C(m + n)x + \beta + b(4Cm + 8Cn - \beta x))\right], \tag{47}
\end{aligned}
$$

$$\frac{E^2}{C} = \frac{k(3 + ax)}{(1 + ax)^2}, \tag{48}$$

8

in terms of elementary functions.

Therefore we have generated a second class of solutions (42)-(48) that models a charged anisotropic star with a linear equation of state. The mass function has the form

$$m(x) = \frac{(2b - k)x^{3/2}}{4C^{3/2}(1 + ax)}.$$

(49)

The form of the mass function (49) represents an energy density which is monotonically decreasing in the stellar interior and remains finite at the centre $x = 0$. It is physically reasonable and has been used in the past to study the properties of isotropic fluid spheres: Matese and Whitman [36] generated equilibrium configurations in general relativity, Finch and Skea [37] studied neutron star models and Mak and Harko [32] analysed anisotropic relativistic stars with this form of mass function. Lobo [10] demonstrated that (49) is consistent with stable dark energy stars which generalises the gravastar model of Mazur and Mottola [38]. It was then shown that large stability regions exist close to the event horizon thereby making it difficult to distinguish dark energy stars from black holes. Sharma and Maharaj [11] found a new class of exact solutions to Einstein equations that can be applied to strange stars with quark matter with this mass distribution. Consequently the mass function (49) is of astrophysical importance in the description of compact objects.

It is interesting to observe that for particular parameter values we can regain uncharged anisotropic and isotropic models ($k = 0$) from our general solution (42)-(48). We regain the following particular cases of physical interest:

3.3.1 Sharma and Maharaj model

If we set $\beta = \alpha \rho_s$ then

$$p_r = \alpha(\rho - \rho_s),$$

where ρ_s is the density at the surface $r = s$. Thus we regain the equation of state of Sharma and Maharaj [11]. Then by setting $C = 1$ and $A^2 D^2 = B$ we find that the line element is of the form

$$\begin{aligned}
ds^2 &= -B(1 + ar^2)^\alpha [1 + (a - b)r^2]^\gamma \exp\left(\frac{-a\beta r^2}{2(a - b)}\right) dt^2 \\
&\quad + \frac{1 + ax}{1 + (a - b)x} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2),
\end{aligned}$$

(50)

where

$$\gamma = \frac{5ab\alpha - 2a^2\alpha - 3b^2\alpha + ab - b^2 + b\beta}{2(a - b)^2}.$$

The line element (50) corresponds to the uncharged anisotropic model of Sharma and Maharaj [11]. They showed that this solution may be used to describe compact objects such as strange stars with a linear equation of state with quark matter.

3.3.2 Lobo model

If we set $\beta = 0$ then

$$p_r = \alpha\rho$$

and we regain the equation of state studied by Lobo [10]. Then on setting $a = 2b, C = 1$ and $A^2 D^2 = 1$ we generate the metric

$$ds^2 = -(1 + br^2)^{(1-\alpha)/2}(1 + 2br^2)^\alpha dt^2 + \left(\frac{1 + 2br^2}{1 + br^2}\right) dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \quad (51)$$

The line element (51) corresponds to the uncharged anisotropic model of Lobo [10]. We point out that the line element (51) serves as an interior solution with $\alpha < -\frac{1}{3}$ which may be matched to an exterior Schwarzschild solution in a model for dark energy stars. Lobo [10] proved that stability regions exist for dark energy stars by selecting particular values of α in a graphical analysis.

3.3.3 Isotropic models

In general $\Delta \neq 0$ so that the model remains anisotropic. However for particular parameter values we can show that $\Delta = 0$ in the relevant limit in the general solution (42)-(48). If we set $a = 0$ and $b = 1$ then we obtain

$$
\begin{aligned}
m &= \frac{\alpha}{2} \\
n &= \frac{1}{4C}[\beta - (1 + 3\alpha)C] \\
\Delta &= \frac{x}{4C(1 - x)}[\beta - 3(1 + \alpha)C][\beta - (1 + 3\alpha)C]. \quad (52)
\end{aligned}
$$

Two different cases arise as a consequence of (52) if we set $\Delta = 0$.

In the first case we observe that when $\beta = 0$ and $\alpha = -1$ then $\Delta = 0$. The equation of state becomes $p_r(= p_t) = -\rho$. In this case the line element becomes

$$ds^2 = -\left(1 - \frac{r^2}{R^2}\right) dt^2 + \left(1 - \frac{r^2}{R^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (53)$$

where we have set $A = D = 1$ and $C = \frac{1}{R^2}$. The metric (53) corresponds to the familiar isotropic uncharged de Sitter model.

In the second case we see that when $\beta = 0$ and $\alpha = -\frac{1}{3}$ then $\Delta = 0$. The equation of state becomes $p_r(= p_t) = -\frac{1}{3}\rho$. In this case the line element becomes

$$ds^2 = -A^2 dt^2 + \left(1 - \frac{r^2}{R^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (54)$$

where we have set $D = 1$ and $C = \frac{1}{R^2}$. The metric (54) corresponds to the well known isotropic uncharged Einstein model.

4 Physical analysis

The solutions found in this paper may be connected to the Einstein-Maxwell equations for the exterior of our source. We need to match the Reissner-Nordstrom exterior spacetime

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right) dt^2 + \left(1 - \frac{2M}{r} + \frac{Q^2}{r^2}\right)^{-1} dr^2 + r^2 \left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

to the interior spacetime (1) across the boundary $r = R$. This generates the conditions

$$1 - \frac{2M}{R} + \frac{Q^2}{R^2} = A^2 y^2 (CR^2)$$

$$\left(1 - \frac{2M}{R} + \frac{Q^2}{R^2}\right)^{-1} = \frac{1 + aCR^2}{1 + (a-b)CR^2}$$

which relates the constants a, b, A, C, D, α and β. This demonstrates that the continuity of the metric coefficients across the boundary of the star $r = R$ is easily satisfied as there are sufficient number of free parameters. If there is a surface layer of charge then the pressure may be nonzero which would place restrictions on the function ν through the matching conditions at the boundary. However the number of free parameters available easily satisfies the necessary conditions that arise for a particular model under investigation.

We now briefly consider the physical behaviour of the models generated in Section 3 for the case $a \neq b$. From the explicit forms (42) and (43) we can easily see that the gravitational potentials $e^{2\nu}$ and $e^{2\lambda}$ are continuous, well behaved and nonsingular at the origin. The energy density ρ is continuous and monotonically decreasing from the centre to the boundary of the star, which is a necessary condition for a realistic model. The radial pressure p_r also has the same feature because ρ and p_r are linked by a linear equation of state. The tangential pressure p_t is also nonsingular at the origin and continuous for a wide range of the parameters a, b and k. To maintain the usual casuality condition we must place the restriction that $0 \leq \alpha \leq 1$ if we require $\frac{dp_r}{d\rho} \leq 1$. However note that our models do allow for $\alpha < 0$ in the case of anisotropic dark energy stars. The form chosen for electric field intensity E is physically reasonable and describes a decreasing function .

With the help of a particular example we can demonstrate the above features graphically. Figures 1-4 represent the energy density, the radial pressure, the tangential pressure and the electric field intensity, respectively. To plot the graphs we choose the parameters $a = 3, b = 2.15, \alpha = 0.33, \beta = \alpha\rho_s = 0.198, C = 1$ and $k = 0.2$, where ρ_s is the density at the boundary $r = s = 1.157$. Note that our choice of $\alpha = 0.33$ ensures that both the radial pressure and the tangential pressure for the neutral sphere vanish

at the boundary. We observe from Figures 1-4 that the matter variables ρ, p_r, p_t and E have the appropriate features to describe a compact relativistic sphere. Solid lines represent uncharged matter and dashed lines include the effect of charge in Figures 1-3. We observe that the effect of E is to produce lower values for ρ, p_r and p_t when compared to the case of uncharged matter. In Figure 5 we have plotted the measure of anisotropy Δ for the same parameter values used above. Note that the effect of the electromagnetic field is to increase the magnitude of Δ which affects the behaviour of p_t.
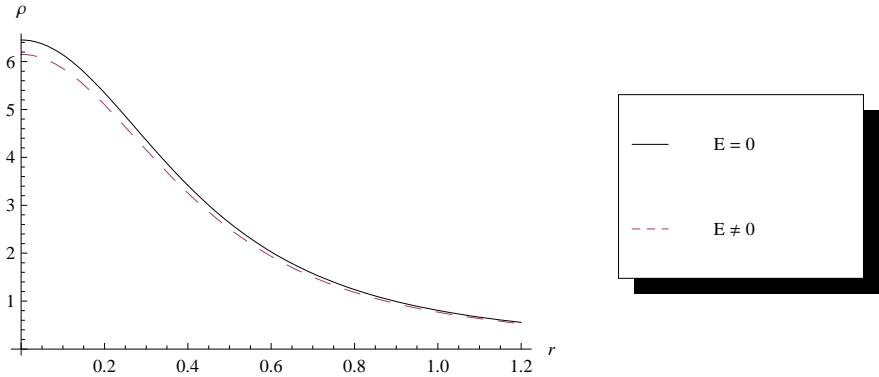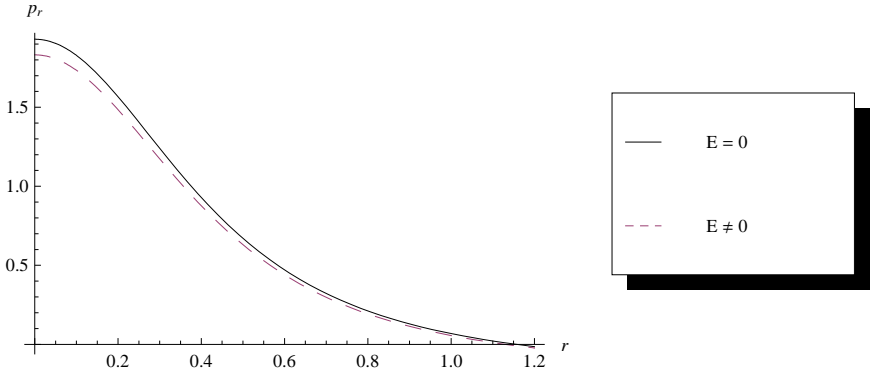

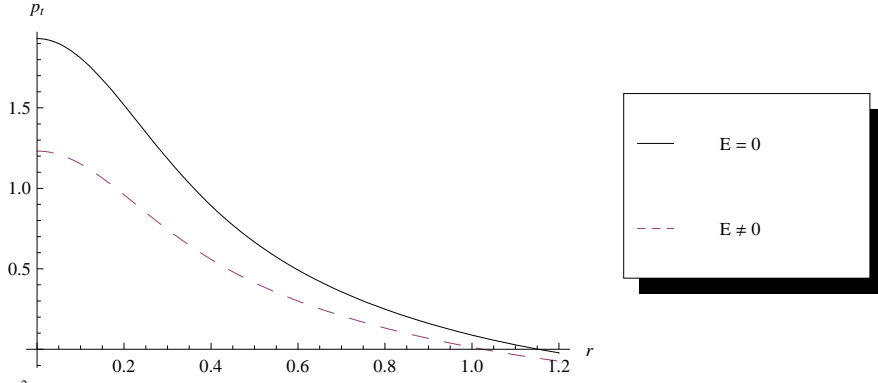Figure 1: Energy density.

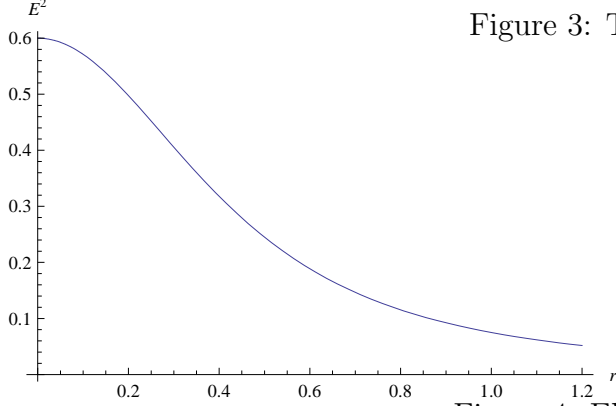Figure 2: Radial pressure.

Figure 3: Tangential pressure.

Figure 4: Electric field intensity.

5 Stellar structure

In this section we show that the solutions generated in this paper can be used to describe realistic compact objects. In particular we seek to compare our results with those of Sharma and Maharaj [11] since they regain values for the stellar mass agreeable with observations. To achieve consistency with Sharma and Maharaj [11] we introduce the transformations

$$\tilde{a} = aR^2, \ \tilde{b} = bR^2, \ \tilde{\beta} = \beta R^2, \ \tilde{k} = kR^2.$$

Under these transformations the energy density becomes

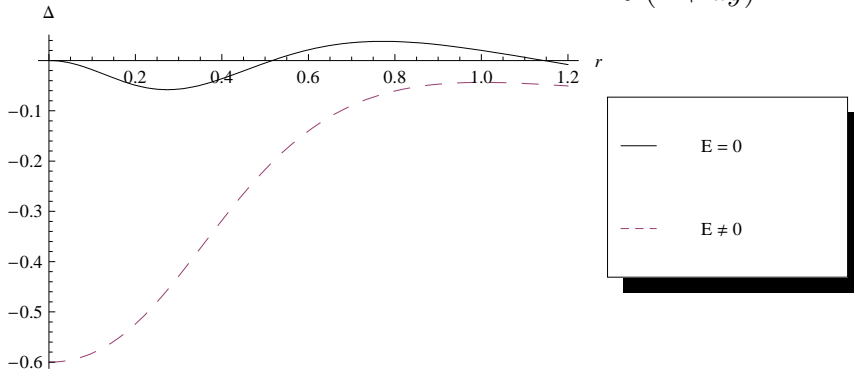$$\rho = \frac{(2\tilde{b} - \tilde{k})(3 + \tilde{a}y)}{2R^2(1 + \tilde{a}y)^2}, \tag{55}$$

Figure 5: Measure of anisotropy.

13

and the mass contained within a radius s has the form

$$M = \frac{(2\tilde{b} - \tilde{k})s^3/R^2}{4(1 + \tilde{a}s^2/R^2)}, \tag{56}$$

where we have set $C = 1$ and $y = \frac{r^2}{R^2}$. When $\tilde{k} = 0$ (or $E = 0$), (55) and (56) reduce to the expressions of Sharma and Maharaj [11]:

$$\rho = \frac{\tilde{b}(3 + \tilde{a}y)}{R^2(1 + \tilde{a}y)^2}, \quad M = \frac{\tilde{b}s^3/R^2}{2(1 + \tilde{a}s^2/R^2)} \tag{57}$$

which gives the density ρ and mass M of an uncharged star of radius s.

If we choose $\tilde{a} = 53.34, \tilde{b} = 54.34, R = 43.245$ km and $s = 7.07$ km then we can produce an uncharged model ($\tilde{k} = 0$) with mass $M = 1.433M_\odot$ and central density $\rho_c = 4.672 \times 10^{15} \text{gcm}^{-3}$. The corresponding value of $\alpha = 0.437$ is obtained by requiring that the anisotropy vanishes at the boundary. To simplify comparison with Sharma and Maharaj [11] we have used the same values of $\tilde{a}, \tilde{b}, R$ and s; however our value for α is a correction. It should be noted that these results are consistent with the equation of state for strange matter formulated by Dey et al [39]. This has astrophysical significance as their model has been used to describe the X-ray binary pulsar SAX J1808.4-3658. When the charge is nonzero we set $\tilde{k} = 37.403$ and then we obtain the mass $M = 0.940M_\odot$ and central density $\rho_c = 3.064 \times 10^{15} \text{gcm}^{-3}$. The values for M and ρ_c generalise the figures of Sharma and Maharaj [11] to include the effect of the electromagnetic field. Choosing different set of values for the parameters will produce different results as shown in Table 1. Note that the values presented in Table 1 correspond to a star of radius $s = 7.07$km. The value of $\tilde{k} = 37.403$ is selected, in generating Table 1, so that the density and mass of the Sharma and Maharaj [11] analysis is regained for uncharged matter. Furthermore, the value of $\tilde{k} = 37.403$ with $E = 0$ generates a star of mass $1.433M_\odot$ which is the same as the strange star model of Dey et al [39]. With this value of $\tilde{k}$ we find that the star has mass $0.940M_\odot$ in the presence of charge so that the stellar core has a lower density which represents a weaker field. This is consistent as the effect of the electromagnetic field is repulsive.

We observe that the values for the mass in the presence of charge ($E \neq 0$) is always less than the uncharged case. The central density of the charged sphere is also less than the uncharged case. Sharma and Maharaj [11] showed that anisotropy affects the mass and central densities of massive objects. We have shown that the inclusion of the electromagnetic field also affects M and ρ_c. Both anisotropy and charge are physical quantities that affect the range of degenerate states in our model. For the calculation of mass and central density we have set $s = 7.07$ km, $R = 43.245$ km, $\tilde{k} = 37.403$ and $\rho_s = 1.17119 \times 10^{15} \text{gcm}^{-3}$ for the uncharged case.

$\tilde{b}$	$\tilde{a}$	α	ρ_c $(\times 10^{15}$ gcm$^{-3})$ $E = 0$	M $(M_\odot)$ $E = 0$	ρ_c $(\times 10^{15}$ gcm$^{-3})$ $E \neq 0$	M $(M_\odot)$ $E \neq 0$
30	23.681	0.401	2.579	1.175	0.971	0.443
40	36.346	0.400	3.439	1.298	1.831	0.691
50	48.307	0.424	4.298	1.396	2.691	0.874
54.34	53.340	0.437	4.671	1.433	3.064	0.940
60	59.788	0.457	5.158	1.477	3.550	1.017
70	70.920	0.495	6.017	1.546	4.410	1.133
80	81.786	0.537	6.877	1.606	5.269	1.231
90	92.442	0.581	7.737	1.659	6.129	1.314
100	102.929	0.627	8.596	1.705	6.989	1.386
183	186.163	1.083	15.730	1.959	14.124	1.759

Table 1: Central density and mass for different anisotropic stellar models for neutral and charged bodies

6 Conclusion

We have found a general framework for the Einstein-Maxwell system of equations with a linear equation of state for anisotropic matter distributions in the presence of the electromagnetic field. Three new classes of exact solutions have been generated to this system of nonlinear equations. We have shown that these classes of solutions satisfy the necessary physical requirements in the description of a charged compact objects with anisotropic matter distribution. We have demonstrated that our models yield stellar structures with masses and densities consistent with the Dey *et al* [38] and Sharma and Maharaj [11] models in the limit of vanishing charge. Therefore it is likely that our solutions may be helpful in the gravitational description of stellar bodies such as SAX J1808.4-3658. The solutions obtained may be useful to model the interior of charged relativistic quark stars with anisotropic matter distribution. Our models contain the uncharged anisotropy models of Sharma and Maharaj [11] and Lobo [10] which describe quark stars and strange matter stars. We believe that the general class of exact solutions found in this paper may assist in more detailed studies of relativistic stellar bodies and allows for different matter distributions because of the form of the linear equation of state chosen.

References

[1] Bowers R L and Liang E P T 1974 *Astrophys. J* **188** 657

[2] Ruderman R 1972 *Ann. Rev. Astron. Astrophys.* **10** 427

[3] Kippenhahn R and Weigert A 1990 *Stellar Structure and Evolution* (Berlin: Springer Verlag)

[4] Sokolov A I 1980 *JETP* **79** 1137

[5] Sawyer R F 1972 *Phys. Rev. Lett.* **29** 382

[6] Binney J and Tremaine J S 1987 *Galactic Dynamics* (Princeton: Princeton University Press)

[7] Herrera L and Santos N O 1995 *Astrophys. J* **438** 308

[8] Letelier P 1980 *Phys. Rev. D* **22** 807

[9] Weber F 1999 *Pulsars as Astrophysical Observatories for Nuclear and Particle Physics* (Bristol: IOP Publishing)

[10] Lobo F S N 2006 *Class. Quantum Grav.* **23** 1525

[11] Sharma R and Maharaj S D 2007 *Mon. Not. R. Astron. Soc.* **375** 1265

[12] Ivanov B V 2002 *Phys. Rev. D* **65** 104001

[13] Thirukkanesh S and Maharaj S D 2006 *Class. Quantum Grav.* **23** 2697

[14] Komathiraj K and Maharaj S D 2007 *J. Math. Phys.* **48** 042501

[15] Sharma R, Mukherjee S and Maharaj S D 2001 *Gen. Relativ. Gravit.* **33** 999

[16] Patel L K and Koppar S K 1987 *Aust. J. Phys.* **40** 441

[17] Tikekar R and Singh G P 1998 *Grav. Cosmol.* **4** 294

[18] Gupta Y K and Kumar M 2005 *Gen. Relativ. Gravit.* **37** 233

[19] Sharma R, Karmakar S and Mukherjee S 2006 *Int. J. Mod. Phys. D* **15** 405

[20] Sharma R and Mukherjee S 2001 *Mod. Phys. Lett. A* **16** 1049

[21] Sharma R and Mukherjee S 2002 *Mod. Phys. Lett. A* **17** 2535

[22] Thomas V O, Ratanpal B S and Vinodkumar P C 2005 *Int. J. Mod. Phys. D* **14** 85

[23] Tikekar R and Thomas V O 1998 *Pramana - J. Phys.* **50** 95

[24] Paul B C and Tikekar R 2005 *Grav. Cosmol.* **11** 244

[25] Mak M K and Harko T 2004 *Int. J. Mod. Phys. D* **13** 149

[26] Komathiraj K and Maharaj S D 2007 *Int. J. Mod. Phys. D* **16** 1803

[27] Witten E 1984 *Phys. Rev. D* **30** 272

[28] Durgapal M C and Bannerji R 1983 *Phys. Rev. D* **27** 328

[29] Dev K and Gleiser M 2002 *Gen. Relativ. Gravit.* **34** 1793

[30] Dev K and Gleiser M 2003 *Gen. Relativ. Gravit.* **35** 1435

[31] Mak M K and Harko T 2002 *Chin. J. Astron. Astrophys.* **2** 248

[32] Mak M K and Harko T 2003 *Proc. Roy. Soc. Lond. A* **459** 393

[33] Chaisi M and Maharaj S D 2005 *Gen. Relativ. Gravit.* **37** 1177

[34] Chaisi M and Maharaj S D 2006 *Pramana - J. Phys.* **66** 609

[35] Maharaj S D and Chaisi M 2006 *Gen. Relativ. Gravit.* **38** 1723

[36] Matese J J and Whitman P G 1980 *Phys. Rev. D* **22** 1270

[37] Finch M R and Skea J E F 1989 *Class. Quantum Grav.* **6** 467

[38] Mazur P O and Mottola E 2004 *Preprint: gr-qc/0405111*

[39] Dey M, Bombaci I, Ray S and Samanta B C 1998 *Phys. Lett. B* **438** 123; addendum: 1999, 447, 352; erratum: 1999, 467, 303