

28 JUL 2015

EASTERN UNIVERSITY SRI LANKA
FACULTY OF COMMERCE AND MANAGEMENT
FIRST YEAR, SECOND SEMESTER EXAMINATION IN
BUSINESS ADMINISTRATION/ COMMERCE 2012/2013
(June/July 2015) – PROPER / REPEAT / RE-REPEAT
COM 1032 BASIC CALCULUS

Answer All Questions

Time: 02 Hours

01. (i) Find the inverse function of $f(x) = \sqrt{x-3}$

(03 marks)

(ii) Given $f(x) = 3x-2$ and $g(x) = \frac{1}{3}x + \frac{2}{3}$, find $g(f(2x))$.

(03 marks)

(iii) For a given function, $f(x) = \begin{cases} x^2 + 5 & \text{if } x < -2 \\ x^2 + 1 & \text{if } x \geq -2 \end{cases}$,

a) specify the domain;

b) sketch the graph;

c) find the values of $f(0)$ and $f\left(-\frac{5}{2}\right)$.

(09 marks)

(iv) Evaluate the following limits:

a)
$$\lim_{x \rightarrow 0} \frac{2 + (-3+x)^2 - 18}{x}$$

b)
$$\lim_{x \rightarrow 4} \frac{x - \sqrt{3x+4}}{4-x}$$

c)
$$\lim_{x \rightarrow 3} \left(\frac{\frac{x}{x+2} - \frac{3}{5}}{x-3} \right)$$

d)
$$\lim_{x \rightarrow \alpha} \frac{x^2 - 5x - 9}{2x^4 + 3x^3}$$

(10 marks)

[Total 25 Marks]

02. (i) Differentiate the following functions with respect to x :

a) $y = 8x^3 - \frac{1}{3x^5} + x - 23$

b) $y = \sqrt[3]{x^2} (2x - x^2)$

c) $y = \ln \left(\frac{(x^2 + 1)^5}{\sqrt{1-x}} \right)$

d) $y = e^{(4-x^3)}$

(15 marks)

(ii) a) Find $\frac{dy}{dx}$ for the following functions.

I) $\frac{y}{x-y} = x^2 + 1$

II) $x^2 y^3 + 3y^2 = x - 4y$

(05marks)

b) If $x = 2t(1+t^2)^{-1}$ and $y = (1-t^2)(1+t^2)^{-1}$, then show that $y \frac{dy}{dx} + x = 0$.

(04 marks)

c) If $y = x(\ln x)^2$, then find $\frac{d^3 y}{dx^3}$, and evaluate it at $x=2$.

(04 marks)

[Total 28 Marks]

03. (i) Find and classify all the critical points for the function,

$$y = x^5 - 15x^3 + 5.$$

(07 Marks)

(ii) If the total cost function of a particular product is given by

$$C(x) = \frac{3}{4}x^2 - 7x + 27, \text{ find the following:}$$

- Average Cost;
- Marginal Cost;
- The level of output for which marginal cost equals average cost.

(06 Marks)

- (ii) A firm has the following demand and the average cost function:

$$x = 480 - 20p \quad \text{and} \quad AC = 10 + \frac{x}{15}.$$

Find:

- the level of output that maximizes the profit;
- the maximum profit;
- the price per unit, when this maximum level is achieved.

(12 Marks)

[Total 25 Marks]

04. (i) Integrate the following:

$$a) \int e^x(e^x - 1) dx \quad b) \int (x^2 + 1)(x + 1)^2 dx.$$

(05 marks)

- (ii) Use substitution method to integrate the following functions:

$$a) \int x^4(2x^5 - 5)^4 dx \quad b) \int x^3 e^{x^4} dx.$$

(05 marks)

- (iii) Evaluate the following:

$$a) \int_1^2 (x+1)(x-1)(2x+1) dx \quad b) \int_{-1}^1 \frac{x^4 + 3x^2 - 1}{x^2} dx.$$

(05 marks)

- (iv) The marginal revenue function of a firm is given by $MR = 240 - x$, where x , the quantity produced. Find the total revenue and the demand function of the firm.

(04 marks)

- (v) The demand function for a commodity is $p = 20 - 2x - x^2$. Find the consumer's surplus when quantity demanded is 3 units.

(03 marks)

[Total 22 Marks]