



EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS
FIRST EXAMINATION IN SCIENCE - 2012/2013
FIRST SEMESTER (Feb./Mar., 2015)
AM 151 - MATHEMATICA
(Proper & Repeat)

Answer all questions

Time : Two hours

1. (a) i. Factor the cubic function $x^3 - 6x^2 + 11x - 6$.
ii. Evaluate the integral $\int_1^3 (1 + 2x + -4x^3)dx$.
iii. Compute $1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{2}}}}$.
iv. Solve the algebraic equation $x^2 - 2x + 1 = 0$.
v. Find the determinant of the matrix $A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 3 \\ 4 & -3 & 8 \end{pmatrix}$, and its inverse if it exists.
- (b) Suppose a curve C is defined by the parametric equation $x = t^2, y = t^3 - 3t$.
i. Plot the curve.
ii. Find the equations of the tangent lines to the curve at the point $(3, 0)$.
iii. Plot the tangent lines at the point $(3, 0)$.

2. (a) Let $f(x) = 6x^3 - 5x^2 - 2x + 1$.
- Evaluate $f(2)$ and $f(1)$.
 - Compute and simplify $\frac{f(1+h) - f(1)}{h}$.
 - Find $\lim_{h \rightarrow 0} \frac{f(1+h) - f(1)}{h}$.
 - Solve $f(x) = 0$.
 - Graph the function $f(x)$ together with the line tangent to the graph of $f(x)$ at the point with x -coordinate equals 1.
- (b)
 - Find all critical numbers for the function $f(x) = x^{4/5}(x-4)^2$.
 - Find the third derivative of the function $g(t) = t^3 - \sqrt{t} + e^{-2t}$.
 - How many numbers of the form $3n^2 + 11$, when n varies from 1 to 2000, are prime?
3. (a) The population $P(t)$ of mosquito larvae growing in a tree hole increases according to the logistics equation with growth constant $k = 0.3$ per day and carrying capacity $A = 1000$.
- Assuming that the initial population of the larvae is 50, find the population $P(t)$ at any time t .
 - After how many days will the larvae population exceed 500?
 - When does the larvae population reach 99% of the maximum capacity?
- (b) Consider the sequence defined by

$$a_n = \frac{4n+1}{3n+2}$$

- Find the first few terms of the sequence.
- Plot the graph of the sequence.
- Find the limit of the sequence.