



**EASTERN UNIVERSITY, SRI LANKA**  
**SECOND EXAMINATION IN SCIENCE - 2013/2014**  
**SECOND SEMESTER (Oct./Nov., 2016)**  
**AM 218 - FIELD THEORY**  
**(PROPER)**

Answer all Questions

Time: Two hours

Q1. State the *Coulomb's law* and *Gauss's Law* in Electric field.

- (a) A total amount of charge  $Q$  is uniformly distributed along a thin, straight, plastic rod of length  $L$ . Find the electric force acting on a point charge  $q$  located at a point  $P$ , at a horizontal distance  $d$  from one end of the rod.
- (b) A spherical conductor of radius  $a$  carrying a charge  $e_1$  is surrounded by a concentric spherical conducting sheet of radius  $b$  and carrying a charge  $e_2$ , both conductors being insulated. Find the potential at a point between the spheres. If the inner conductor is connected by a fine insulated conducting wire passing through a small hole in the outer conductor to a distant uncharged and insulated spherical conductor of radius  $c$ , prove that the latter will be raised to a potential  $\frac{e_1 b + e_2 a}{4\pi\epsilon_0 b(a+c)}$ , where  $\epsilon_0$  is the permittivity of free space.

Q2. (a) Define the terms *electric potential* and *electric dipole*.

A total charge  $Q$  is distributed along a straight rod of length  $L$ . Find the potential at a point  $P$  at a vertical distance  $h$  from the mid point of the rod.

Prove that the electric potential  $\phi$  at a point  $P$  with position vector  $\underline{r}$  from the dipole moment  $\underline{p}$  is given by

$$\phi = \frac{\underline{p} \cdot \underline{r}}{4\pi\epsilon_0 r^3}$$

(b) State the Poisson's equation in electric field.

Show that the solution of the equation  $\nabla^2\phi = 0$  in rectangular coordinates given by

$$\phi = e^{\pm i\alpha x} e^{\pm i\beta y} e^{\pm \sqrt{\alpha^2 + \beta^2} z},$$

where  $\alpha$  and  $\beta$  are arbitrary constants.

Q3. (a) Using Ampere's circuit law and Biot-Savart law, prove that  $\nabla^2\phi = 0$  is scalar potential.

(b) Show that the equivalence between Biot-Savart and Ampere's laws is brought out by determining the magnetic field  $\vec{B}$  due to an infinite conductor carrying a steady current through it.

(c) Particle  $A$  with charge  $q$  and mass  $m_A$ , and particle  $B$  with charge  $q$  and mass  $m_B$  are accelerated from rest by a uniform magnetic field in circular paths. The radii of the trajectories of the particles  $A$  and  $B$  are  $R$  and  $2R$ , respectively. The direction of the magnetic field is perpendicular to the velocity of the particle. Show that  $m_A : m_B = 1 : 8$ .

Q4. (a) Define the terms magnetic flux density and the magnetic dipole. Show that  $\vec{\nabla} \cdot \vec{B} = 0$  in space, where  $\vec{B}$  is the magnetic field.

(b) If the magnetic field normal to the plane of a circular coil of  $n$  turns and radius  $r$  which carries a current  $I$  is measured on the axis of the coil, show that the magnetic field at a small distance  $h$  from the center of the coil is

$$\frac{\mu_0 n I}{2r} \left( 1 - \frac{3h^2}{2r^2} \right) \quad \text{where } r \gg h.$$

(c) An amount of charge  $Q$  is uniformly distributed over a disk of radius  $R$ . The disk spins about its axis with angular velocity  $\omega$ . Find the magnetic moment of the disk.