EASTERN UNIVERSITY, SRI LANKA

DEPARTMENT OF MATHEMATICS

THIRD YEAR EXAMINATION IN SCIENCE - 2013/2014

SECOND SEMESTER (June,2016)

AM 307 - CLASSICAL MECHANICS

SPECIAL REPEAT

Answer all Questions — Time: Three hours

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1. Two frames of reference S and S' have a common origin O and S' rotates with an constant angular velocity $\underline{\omega}$ relative to S. If a moving particle P has its position vector as $\underline{r}$ relative to O at time t, show that :

   (a) $\dfrac{d\underline{r}}{dt} = \dfrac{\partial \underline{r}}{\partial t} + \underline{\omega} \wedge \underline{r}$, and

   (b) $\dfrac{d^2\underline{r}}{dt^2} = \dfrac{\partial^2 \underline{r}}{\partial t^2} + 2\underline{\omega} \wedge \dfrac{\partial \underline{r}}{\partial t} + \dfrac{\partial \underline{\omega}}{\partial t} \wedge \underline{r} + \underline{\omega} \wedge (\underline{\omega} \wedge \underline{r})$.

   An object is thrown downward with an initial speed v_0. Prove that after time t the object is deflected east of the vertical by the amount

$$\omega v_0 \sin\lambda t^2 + \frac{1}{3}\omega g \sin\lambda t^3,$$

   where λ is the earth's co - latitude.

2. (a) With the usual notations, obtain the equations of motion for a system

particles in the following forms:

i. $M\underline{f}_G = \sum_{i=1}^{N} \underline{F}_i,$

ii. $\dfrac{d\underline{H}}{dt} = \sum_{i=1}^{N} \underline{r}_i \wedge \underline{F}_i,$

where $\sum_{i=1}^{N} \underline{h}_i = \underline{H}$ and $\underline{h}_i = \underline{r}_i \wedge m_i \underline{v}_i$.

(State clearly the results that you may use)

(b) A solid of mass M is in the form of a tetrahedron $OXYZ$, the edges OX, O of which are mutually perpendicular, rests with XOY on a fixed smooth izontal plane and YOZ against a smooth vertical wall. The normal t rough face XYZ is in the direction of a unit vector $\underline{n}$. A heavy uniform s of mass m and center C rolls down the face causing the tetrahedron to ac a velocity $-V\underline{j}$ where $\underline{j}$ is the unit vector along OY

If $\overrightarrow{OC} = \underline{r}$, then prove that

$$(M + m)V - m\dot{\underline{r}} \cdot \underline{j} = \text{constant}$$

and that

$$\frac{7}{5}\,\ddot{\underline{r}} = \underline{f} - \underline{n}(\underline{n} \cdot \underline{f})\,,$$

where $\underline{f} = \underline{g} + \dot{V}\underline{j}$ and $\underline{g}$ is the acceleration of gravity.

6.

3. With the usual notation obtain the Euler's equations for the motion of the body, having a point fixed, in the form:

$$A\dot{\omega}_1 - (B - C)\omega_2\omega_3 = N_1,$$

$$B\dot{\omega}_2 - (C - A)\omega_1\omega_3 = N_2,$$

$$C\dot{\omega}_3 - (A - B)\omega_1\omega_2 = N_3.$$

A body moves about a point O under no forces. The principle moment of in at O being $3A$, $5A$ and $6A$. Initially the angular velocity has components ω_1 $\omega_2 = 0$, $\omega_3 = 3$ about the corresponding principal axes. Show that at time t,

$$\omega_2 = \frac{3n}{\sqrt{5}} \tan\left(\frac{nt}{\sqrt{5}}\right).$$

4. Obtain the Lagrange's equations of motion using D'Alembert's principle for a conservative holonomic dynamical system.

A sphere of mass M and radius R rolls without slipping down the inclined plane of a wedge shaped block of mass m that is free to move on a frictionless horizontal surface.

(a) Find the Lagrange's equations for this system subject to the force of gravity at the surface of the earth, given that all objects are initially at rest and the center of the sphere is at a distance H above the surface.

(b) Find the motion of the system by integrating Lagrange's equations.

5. (a) Define *Hamiltonian* function in terms of Lagrangian function .

Show that, with the usual notations, that the Hamiltonian equations are given by

$$\dot{q}_j = \frac{\partial H}{\partial p_j}, \ \dot{p}_j = -\frac{\partial H}{\partial q_j} \text{ and } \frac{\partial H}{\partial t} = -\frac{\partial L}{\partial t}.$$

(b) Prove that if the time t does not occur in Lagrangian function L, then the Hamiltonian function H is also not involved in t.

(c) Write down the Hamiltonian function H and then find the equation of motion for a simple pendulum.

6. (a) Define the poisson bracket.

Show that for any function $f(q_i, p_i, t)$,

$$\frac{df}{dt} = \frac{\partial f}{\partial t} + [f, H],$$

where H is a Hamiltonian function.

(b) With the usual notations, prove that:

i. $\frac{\partial}{\partial t}[f, g] = \left[\frac{\partial f}{\partial t}, g\right] + \left[f, \frac{\partial g}{\partial t}\right]$;

ii. $[f, q_k] = -\frac{\partial f}{\partial p_k}$;

iii. $[f, p_k] = \frac{\partial f}{\partial q_k}$.

(c) Show that, if F and g are constant of motion then their poisson bracket $[f, g]$ is a constant of motion.