

# MARKOV-BERNSTEIN INEQUALITIES

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By

**Kiddnan Kobindarajah**

512.97072  
KOB  
PR



Under the Supervision of

**Professor D S Lubinsky**



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## Abstract

Markov-Bernstein inequalities have a vital role in approximation theory, probability, statistics, and etc. Many theorems of the theory of approximation depend upon the fact that a polynomial of degree  $n$  cannot change too rapidly; in other words, its derivative cannot be too large. Markov-Bernstein and related inequalities give estimates for the magnitude of the derivative, as compared to the polynomial itself. The importance of these inequalities have attracted many mathematicians to further their researches in this area and thus there are many generalizations and new developments.

In this dissertation we prove  $L_p$  Markov-Bernstein inequalities on all arcs of the circle. In the first chapter we present a wide survey on Markov-Bernstein and related inequalities and in the second chapter we prove our new results. Chapter 3 presents a further survey on Markov-Bernstein inequalities for nonnegative generalized polynomials. Finally in Chapter 4 we extend the results obtained in Chapter 2 to nonnegative generalized polynomials.

The results in Chapter 2 have been submitted to the Journal of Approximation Theory as a joint paper with Professor D.S.Lubinsky. The results from Chapter 4 have been submitted to the Proceedings of the Tenth International Conference on Approximation Theory in the USA 2001.

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