EASTERN UNIVERSITY, SRI LANKA

FIRST EXAMINATION IN AGRICULTURE

1993/94 (MAY' 2001)

REPEAT

EXTERNAL DEGREE

BASIC MATHEMATICS

Answer five questions only

Time : 03 hours

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1. (a) Simplify the following:

   i. $\left(\frac{125}{64}\right)^{\frac{1}{3}} \times \left(\frac{25}{64}\right)^{-\frac{1}{2}} \times 3^0$;

   ii. $\left(\frac{a^{\frac{5}{3}}\, b^{-\frac{2}{3}}\, c^{\frac{1}{3}}}{b^{\frac{3}{2}}\, c^{-\frac{1}{6}}}\right)^{-\frac{1}{2}} \times \left(\frac{a^{\frac{4}{3}}\, b^{-\frac{1}{2}}}{a^{-\frac{2}{3}}\, c^{\frac{1}{2}}}\right)^{\frac{3}{2}}$;

   iii. $\frac{2x-3}{4} - \frac{3x-5}{6} = \frac{x+4}{12} - \frac{1}{3}$;

   iv. $\frac{\log_a \sqrt{27} + \log_a \sqrt{8} - \log_a \sqrt{125}}{\log_a 6 - \log_a 5}$.

   (b) Solve the following equations :

   i. $6 \log 3 + 4 \log x - \log 9 = 2 \log 25$;

   ii. $\log_2 8 + \log_3 27 = \log_2 x + \log_5 25$.

(c) If $m = a^x$, $n = a^y$ and $m^y n^x = a^{2(x+y)}$, show that $\frac{1}{x} + \frac{1}{y} = 1$.

2. Factorize the following expressions:

(a) $64x^2 - 100y^2$;

(b) $ax^2 - 4ax - 21$;

(c) $8ax - 3by - 6ay + 4bx$;

(d) $3a(p - q) - 2b(q - p)$;

(e) $2(x - 1)^2 - 3(x - 1)$;

(f) $(2x + 5y)^2 - (2x - 5y)^2$.

3. (a) Determine the nature of the roots of the following equations:

i. $x^2 - 8x + 16 = 0$;

ii. $x^2 + 6x - 7 = 0$;

iii. $4x^2 - 12x + 9$.

(b) Solve the following equations:

i. $\frac{x+2}{2x+1} = \frac{6}{x+3}$;

ii. $x^2 - 4x - 7 = 0$.

(c) Solve the following exponential equations:

i. $7^{2x} - 7^{x+2} + 48 = 0$;

ii. $2^{2x} \times 4^{2x-3} = 8^{-2x}$;

iii. $\sqrt{3^x} + \sqrt{3^{2-x}} - 6 \times \sqrt{3^{-x}} = 0$.

4. (a) Find the slope of the line through the points $(0, -2)$, $(2, 4)$.

   (b) Find the equation of the line through the given points $(-2, 5), (-3, -4)$.

   (c) Find the equation of the straight line passing through the point $(0, 2)$ and is parallel to the line $y = 8x$.

   (d) Find the equation of the line which passing through the point of intersection of the lines $2x - 3y = 13$ and $5x + 2y = 4$ and perpendicular to the straight line $3x + 4y = 0$.

5. (a) Express the following in terms of the trignometrical ratios of acute angles.

   i. $\sin 105°$;

   ii. $\cos(-105)°$;

   iii. $\tan 165°$;

   iv. $\cos 135°$.

   (b) Solve the equation, $2\sin^2\theta + \sqrt{3}\cos\theta + 1 = 0$ for the values of θ from $0°$ to $360°$ inclusive.

   (c) Prove the following:

   i. $\dfrac{1 + \sin x}{\cos x} + \dfrac{\cos x}{1 + \sin x} = 2\sec x$;

   ii. $\tan^2 x - \sin^2 x = \sin^4 x \ \sec^2 x$.

   (d) Find, without using tables or calculators, the value of $\cos\ (120° + 45°)$.

6. (a) Evaluate the following;

i. $\lim_{x\to 1} \dfrac{x^2-3x+4}{x^2-x+3}$;

ii. $\lim_{x\to \infty} \dfrac{2x^2+x-5}{3x^2-2x}$;

iii. $\lim_{x\to 0} \dfrac{\sqrt{1-x}-1}{x}$;

iv. $\lim_{x\to 9} \dfrac{\sqrt{x}-3}{x-9}$;

v. $\lim_{x\to -2} \dfrac{x^3+8}{x+2}$.

(b) Differentiate the following with respect to x by using the first principle.

i. $y = x^2$.

ii. $y = \dfrac{1}{2+3x}$.

7. (a) Differentiate the following functions with respect to x.

i. $y = \ln\left(\dfrac{x^2+2x}{x^3-3x+4}\right)$;

ii. $y = (x^2+2x+4)e^x$.

(b) i. If $x = \dfrac{t}{1-t}$ and $y = \dfrac{t^2}{1-t}$, then find $\dfrac{dy}{dx}$ in terms of t.

ii. If $y = x^2 + \dfrac{1}{x}$ then prove that

$$x^2\frac{d^2y}{dx} + x\frac{dy}{dx} = 4y.$$

(c) Find the maximum and minimum points of the func
$x^3 - 3x^2 + 5$.

8. (a) Evaluate the following indefinite integrals with respe

i. $\int \frac{x^4 - x^3 + x - 1}{x^3 - x^2}\, dx;$

ii. $\int \frac{e^x}{1 + e^x}\, dx;$

iii. $\int \frac{dx}{x\sqrt{\log x}};$

iv. $\int \frac{3x^3}{x^2 + 2}\, dx.$

(b) Evaluate the following:

i. $\int_1^2 (x^2 + 3x - 5)\, dx;$

ii. $\int_2^3 \frac{x}{x + x^2}\, dx;$

iii. $\int_2^3 \frac{x}{\sqrt{x^2 - 3}}\, dx.$