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EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE 1994/95 & 95/96

(August/September'97) - RE-REPEAT

MT205 & 208 - MATHEMATICAL METHODS & NUMERICAL ANALYSIS

Time: Two hours.

Answer only four questions selecting two from each section.

SECTION A

MATHEMATICAL METHODS

1. (a) i. Write the transformation equations for the following tensors.

$$A_{jk}^{i}$$
, B_{l}^{ijk} , C_{mn} .
ii. If $\overline{A}_{r}^{p} = \frac{\partial \overline{x}^{p}}{\partial x^{q}} \frac{\partial x^{s}}{\partial \overline{x}^{r}} A_{s}^{q}$, then prove that $A_{s}^{q} = \frac{\partial x^{q}}{\partial \overline{x}^{p}} \frac{\partial \overline{x}^{r}}{\partial x^{s}} \overline{A}_{r}^{p}$.

- (b) If $A(p,q,r)B_r^{qs}=\mathcal{C}_p^s$, where B_r^{qs} is an arbitrary tensor and \mathcal{C}_p^s a tensor, then prove that A(p,q,r) is a tensor.
- (c) Find the covariant and contravariant components of a tensor in cylindrical coordinates (ρ, ϕ, z) if its covariant components in rectangular co-ordinates are $xy, 2y z^2, xz$.
- 2. (a) Define the following:
 - i. Christoffel symbols of first and second kind,
 - ii. Geodesics.

(b) Prove the following:

i.
$$[pq, r] = [qp, r],$$

ii.
$$\Gamma^s_{pq} = \Gamma^s_{qp}$$
,

iii.
$$[pq, r] = g_{rs} \Gamma^s_{pq}$$
.

and corresponding Geodesic

- (c) Find the second kind of the Christoffel symbol in spherical co-ordinates (r, θ, ϕ) .
- 3. (a) i. Explain the term "Covariant derivative" as applied to a tensor of type A^a_{bc} .
 - ii. Write the covariant derivative with respect to x^q of each of the following tensors.

$$A_l^{jk}, \quad A_{klm}^j, \quad A_{lmn}^{jk}.$$

(b) With the usual notations, prove the following:

i.
$$\frac{\partial g_{pq}}{\partial x^m} = [pm, q] + [qm, p];$$

ii.
$$\frac{\partial g^{pq}}{\partial x^m} = -g^{pn} \Gamma^q_{mn} - g^{qn} \Gamma^p_{mn}$$
.

Deduce that the covariant derivatives of the tensors g_{jk} , g^{jk} , δ_k^j are zero.

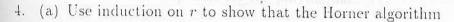
(c) Using the covariant derivative of metric tensor, prove that

fixed point
$$\Gamma_{ca}^e = \frac{1}{2} g^{eb} \left[\partial_c(g_{ab}) + \partial_a(g_{cb}) - \partial_b(g_{ca}) \right],$$

where
$$\partial_i = \frac{\partial}{\partial x^i}$$
, and the number $\frac{\pi}{2} = (1.921 \, R/321 \, M_{\odot})$

SECTION B

NUMERICAL ANALYSIS



$$k_0 = a_n,$$

 $k_r = b k_{r-1} + a_{r-1}$ for $r = 1, 2, ..., n$,

generates a sequence (k_r) that satisfies

$$k_r = a_n b^r + a_{n-1}b^{r-1} + \cdots + a_{n-r}$$
 for $r = 0, 1, \ldots n$.

Show also that

$$a_n x^n + a_{n-1} x^{n-1} + \ldots + a_0 = (x-b)(k_0 x^{n-1} + k_1 x^{n-2} + \cdots + k_{n-1}) + k_n.$$

Hence find the following:

- i. the quotient polynomial and remainder when $p(x) = 3x^5 + 5x^4 + 8x^2 + 7x + 4 \text{ is devided by } x + 2;$
 - ii. the Taylor series of p(x) about the point x = -2.
- (b) Explain what is meant by
 - i. fixed point representation,
 - ii. floating point representation.

For base 16, round the number $\frac{\pi}{2} = (1.921FB54...)_{16}$ to

- i. 5 places in fixed point,
- ii. 5 digits in floating point.

(In base 16,
$$A = 10$$
, $B = 11$, $C = 12$, $D = 13$, $E = 14$, $F = 15$.)

- 5. Let f_0, f_1, \ldots, f_m be a sturm sequence of polynomial of decreasing degree with the properties
 - (a) $f_m(x) \neq 0$ for any x in [a, b],
 - (b) if $f_r(\alpha) = 0$ $(1 \le r < m 1)$, then $f_{r-1}(\alpha) f_{r+1}(\alpha) < 0$,
 - (c) if $f_0(\alpha) = 0$, then $f_1(\alpha)f'_0(\alpha) > 0$; where $\alpha \in [a, b]$

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Let V(x) denote the number of sign charges in this sequence at any fixed value of x (for which $f_0(x) \neq 0$). Prove that f_0 has V(a) - V(b) zeros in the interval [a, b].

T 205 - Pascal Programming

Construct a sturm sequence for the polynomial

$$P(x) = x^4 + 2x - 1.$$

Show that

- (a) only two roots of P are real; with the comments chains of
- (b) one root is positive and the other is negative;
- (c) the positive root lies in the interval [0, 1].
- 6. Define what is meant by the statement that a function $g:[a,b] \longrightarrow [a,b]$ is a contraction mapping on [a,b].

Let g be a contraction mapping on [a,b] with Lipschitz constant k. Show that there is a unique $c \in [a,b]$ such that c-g(c)=0. Show also that c is the limit of the sequence $\{x_n\}$ given by

$$x_{n+1} = g(x_n), \quad n = 0, 1, \dots, \text{ for any } x_0 \in [a, b].$$

Prove that
$$|x_1 - c| \le \frac{k}{1-k}|x_1 - x_0|$$

Show that there exists a unique $c \in [0, 1]$ such that c = g(x) where $g(x) = \frac{1}{2}e^{x-1}$. Use the above inequality to find lower and upper bound for c, taking $x_0 = 0.5$.