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Eastern University, Sri Lanka

EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE - 1994/95 & 95/96

(Aug./Sep.'97)

MT207 & 209 - CLASSICAL MECHANICS II AND DIFFERENTIAL EQUATIONS & FOURIER SERIES

Time Allowed: 02 Hours.

Answer only four questions selecting two from each section.

SECTION A

CLASSICAL MECHANICS II

1. With usual notations, obtain the following equations for a common catenary

   (a) $s = c\tan\psi$,

   (b) $y = c\sec\psi$,

   (c) $T = \omega y$,

   (d) $y^2 = s^2 + c^2$.

   A uniform chain of length l and weight W_1 hangs between two fixed points at the same level and a weight W_2 is attached at the mid point of the chain. If the sag at the middle is d, show that the tension of the chain at each fixed point is

$$\left(\frac{d}{2l} + \frac{l}{8d}\right) W_1 + \frac{l}{4d} W_2.$$

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2. State the Bernoulli-Euler law of flexure.

A beam AB of length a is clamped horizontally at A and is loaded so that the load intensity at any point P is proportional to AP^2. If the total load supported is W, then prove that the vertical force applied at B required to hold B at the same level as A is $\frac{13}{20}W$.

3. With the usual notations, prove the Claypeyron's equation

$$M_1 a + 2M_2(a+b) + M_3 b = -\frac{\omega}{4}(a^3 + b^3) + 6EI\left(\frac{y_a}{a} + \frac{y_b}{b}\right)$$

for the moment of a slightly elastic beam.

A uniform rod ABC of weight ω per unit length is supported at its ends A, C and its point B on its length. The three points A, B and C are at the same horizontal level such that $AB = a$ & $BC = b$. Show that the reaction of the support at A is

$$\frac{\omega}{8a}(3a^2 + ab - b^2).$$

Hence show that the rod can remain in contact with all the supports if

$$\frac{\sqrt{13}-1}{6} < \frac{a}{b} < \frac{\sqrt{13}+1}{2}.$$

SECTION B

DIFFERENTIAL EQUATIONS & FOURIER SERIES

4. Obtain solution of the differential equation

$$x(1-x)\frac{d^2y}{dx^2}-(1+3x)\frac{dy}{dx}-y=0$$

in series.

5. (a) State the necessary and sufficient condition for the equation $Pdx+Qdy+Rdz=0$ to be integrable, where P, Q, R are functions of x, y and z.

Test the integrability of the differential equation

$$yz\,dx+(xz-yz^3)\,dy-2xy\,dz=0,$$

and solve this when it is integrable.

(b) Find the general solution of each of the following:

i. $\dfrac{dx}{y^3x-2x^4}=\dfrac{dy}{2y^4-x^3y}=\dfrac{dz}{9z(x^3-y^3)}$,

ii. $\dfrac{dx}{x(2y^4-z^4)}=\dfrac{dy}{y(z^4-2x^4)}=\dfrac{dz}{z(x^4-y^4)}$.

(c) Find the complete solution and singular solution of each of the following equations if $p=\dfrac{\partial z}{\partial x}$ & $q=\dfrac{\partial z}{\partial y}$.

i. $p^2+pq=4z$,

ii. $x^4p^2-yzq-z^2=0$.

(Hint: Use $X=\frac{1}{x}$, $Y=\ln y$, $Z=\ln z$.)

6. (a) Prove that for $0 \leq x \leq \pi$,

$$\sin x = \frac{4}{\pi}\left\{\frac{1}{2} - \frac{1}{1.3}\cos 2x - \frac{1}{3.5}\cos 4x - \frac{1}{5.7}\cos 6x - \ldots\ldots\right\}.$$

(b) Define the finite Fourier sine transform of $F(x)$, $0 < x < l$, and write down its inversion formula.

The potential $V(x,t)$ at any point of a cable of resistance R and capacitance C per unit length satisfies the equation

$$\frac{\partial^2 V}{\partial x^2} = \frac{1}{K}\frac{\partial V}{\partial t},$$

where $K = \frac{1}{RC}$.

Determine the potential $V(x,t)$ for the cable of length l if both ends of the cable are earthed and if

$$V(x,0) = \begin{cases} 2v_0\,\dfrac{x}{l} & \text{for } \ 0 \leq x \leq \dfrac{l}{2}, \\ 2v_0\left(1 - \dfrac{x}{l}\right) & \text{for } \ \dfrac{l}{2} < x \leq l. \end{cases}$$