EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS
SECOND EXAMINATION IN SCIENCE -2008/2009
SECOND SEMESTER (Sept./Oct., 2010)
MT 205 - DIFFERENTIAL GEOMETRY
(PROPER & REPEAT)

Answer all Questions **Time: One hour**

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1. (a) State the *Frenet – Serret* formula.

   If $\underline{r} = \underline{r}(s)$ is the position vector of a point P with arc-length s as a parameter on a curve C, then show that:

   i. $\underline{r}'' \cdot \underline{r}'' = \kappa^2$;

   ii. $[\underline{r}', \underline{r}'', \underline{r}'''] = \kappa^2\tau$;

   where $\underline{r}' = \dfrac{d\underline{r}}{ds}$, κ is the curvature and τ is the torsion of the curve C.

   (b) Show that the curve

   $$\underline{r}_1 = -\frac{1}{\tau}\underline{n} + \int \underline{b} ds$$

   has constant curvature $\pm\,\tau$ when the curve $\underline{r} = \underline{r}(s)$ has constant torsion τ.

   (c) Let C be a curve with constant torsion at any point P on the curve. Point Q is taken at a constant distance c from the point P on the binormal to the curve C at P. Show that the angle between the binormal to the locus of Q and the binormal of the given curve is

   $$\tan^{-1}\left(\frac{c\tau^2}{\kappa\sqrt{1+c^2\tau^2}}\right).$$

2. Define the terms *involute* and *evolute* of a curve.

(a) With the usual notations show that the equation of *involute* of the curve $C : \underline{r} = \underline{r}(s)$ is given by

$$\underline{R} = \underline{r} + (c - s)\underline{t},$$

where c is a constant.

(b) Find the *involute* and *evolute* of the cubic curve given by

$$\underline{r}(u) = (3u, 3u^2, 2u^3).$$