EASTERN UNIVERSITY, SRI LANKA
DEPARTMENT OF MATHEMATICS
THIRD EXAMINATION IN SCIENCE 2008/2009
SECOND SEMESTER (Sept/Oct., 2010)
MT 307 - CLASSICAL MECHANICS III
(PROPER & REPEAT)

Answer all questions . Time: Three hours

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1. Define the following terms:

   (a) linear momentum;

   (b) angular momentum;

   (c) moment of force.

   A uniform circular disc of mass M and radius R is mounted, that it can turn freely about its center, which is fixed. It is spinning with angular velocity $\underline{\omega}$ about the perpendicular to its plane at the center, the plane being horizontal. A particle of mass m, falling vertically, hits the disc near the edge and adheres to it. Prove that immediately afterwards the particle is moving in a direction inclined to the horizontal at an angle α given by the equation

$$\tan\alpha \;=\; \left(\frac{4m(M+2m)}{M(M+4m)}\right)\left(\frac{v}{R\omega}\right),$$

   where v is the speed of the particle just before impact and $\omega = |\underline{\omega}|$.

2. (a) With the usual notations from the equation of motion derive the equation for kinetic energy in the form of,

$$\frac{dT}{dt} = \underline{F}.\underline{v}$$

for a single particle with a constant mass.

If the mass varies with time then show that the corresponding equation is

$$\frac{d\,(mT)}{dt} = \underline{F}.\underline{p}\,.$$

(b) A uniform sphere of mass M and radius a is released from rest on a plane inclined at an angle α to the horizontal. If the sphere rolls down without slipping, show that the acceleration of the center of the sphere is constant and is equal to $\frac{5}{7}\,g\sin\alpha$.

3. With the usual notations, obtain the Euler's equations of motion for a rigid body having a point fixed, in the following form:

$$A\dot{\omega}_1 - (B-C)\omega_2\omega_3 = M_1,$$

$$B\dot{\omega}_2 - (C-A)\omega_1\omega_3 = M_2,$$

$$C\dot{\omega}_3 - (A-B)\omega_1\omega_2 = M_3.$$

The principal moments of inertia of a body at the center of mass are A, $3A$, $6A$. The body is rotated that it's angular velocities about the axis are $3n$, $2n$, n respectively. If in the subsequent motion under no force and ω_1, ω_2, ω_3 denote the angular velocities about the principal axes at the time t then show that

$$\omega_1 = 3\omega_3 = \frac{9n}{\sqrt{5}}\,\text{sech}\,u \quad \text{and} \quad \omega_2 = 3n\tanh u,$$

where $u = 3nt + \ln\sqrt{5}$.

4. Obtain the Lagrange's equations of motion using D'Alembert's principle for a holonomic system.

Two mass points of mass m_1 and m_2 are connected by a string passing through a hole in a smooth table so that m_1 rests on the table and m_2 hangs suspended.

(a) Assuming m_2 moves only in a vertical line, find the generalized coordinates for the system.

(b) Write down the Lagrange's equations for the system and if possible discuss the physical significance any of them might have.

(c) Reduce the problem to a single second order differential equation and obtain a first integral of the equation.

5. (a) Define the Hamiltonian interms of the Lagrangian.

Hence show that the Hamiltonian's equations are given by

$$\dot{q}_j = \frac{\partial H}{\partial p_j}\ , \qquad \dot{p}_j = -\frac{\partial H}{\partial q_j},$$

when H does or does not contain the variable time t explicitly.

(b) If the Hamiltonian H is independent of time t explicitly, then prove that it is a constant, and equal to the total energy of the system.

(c) A block of mass m that can slide, without friction, along an inclined plane surface of the heavy wedge (mass m'). The wedge is free to move, also without friction, along a horizontal surface.

i. Calculate the Hamiltonian function H; find out whether it is conserved.

ii. Calculate the energy E; is $E = H$?; is energy conserved?

6. (a) Define the Poisson Bracket.

With the usual notations show that

$$\frac{dF}{dt} = [F, H] + \frac{\partial F}{\partial t}$$

for a function $F = F(p_j, q_j, t), \quad j = 1, 2, \cdots, n$. Prove the Poisson's theorem that $[F, G]$ is a constant of motion when $F = F(p_j, q_j, t)$ and $G = G(p_j, q_j, t)$, $j = 1, 2, \cdots, n$ are constant of motion.

(b) Find the frequency of oscillation of a particle of mass m which is moving along a line and is attached to spring whose other end is fixed at a point A at a distance l from the line.