EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE - 2005/2006

(Mar./Apr.' 2008)

SECOND SEMESTER

ST 204 - STATISTICAL INFERENCE II

(Repeat)

Answer all questions

Time : Two hours

(a) Let $X_1, X_2, \cdots, X_n$ be independent random samples from normal population with mean μ and variance σ^2. Show that,

i. the statistic $\hat{\mu} = \frac{1}{n+1}\sum_{i=1}^{n} X_i$ is biased for μ.

ii. $S^2 = \frac{1}{n-1}\sum_{i=1}^{n}(X_i - \bar{X})^2$ is an unbiased estimator for σ^2.

(b) Two methods for teaching reading were applied to two randomly selected groups of elementary school children and compared on the basis of a reading comprehension test given at the end of learning period. The sample means and variances computed from the test scores in the accompanying table below. Do the data present sufficient evidence to indicate a difference in the mean scores for the populations associated with the two teaching methods ? Test at $\alpha = 0.05$ level of significance.(Only assume the two populations are normal)

	Method 1	*Method* 2
No. of children		
in group	11	14
$\bar{Y}$	64	69
s^2	52	71

Q2. (a) Define Type I error, Type II error and unbiased estimator.

(b) Let $X_1, X_2, \cdots, X_n$ be a random sample from a normal population with parameters μ and $\sigma^2 = 4$.

The test is $H_0 : \mu = 0$ Vs $H_1 : \mu = 1$. The critical region is given by $\{\underline{X} \,/ \sum_{i=1}^{n} X_i > k\}$

If $\alpha = \beta = 0.01$ then find the critical region.

Where $\alpha = P(Type\ I\ error)$ and $\beta = P(Type\ 2\ error)$.

(c) A machine in a certain factory must be repaired if it produces more than 10% defectives among the large lot of items it produces in a day. A random sample of 100 items from the day's production contains 15 defectives, and the foreman says that the machine must be repaired. Does the sample evidence support his decision ? Use $\alpha = 0.01$

Q3. (a) State the Neymann-Pearson lemma and the Likelihood Ratio Test

(b) Let $Y_1, Y_2, \cdots, Y_n$ be a random sample from the probability density function given by,

$$f(y) = \begin{cases} \frac{1}{\theta} m y^{m-1} e^{-(y^m/\theta)}, & y > 0; \\ 0, & \text{elsewhere.} \end{cases}$$

with m denoting a known constant.

i. Find the uniformly most powerful test for testing $H_0 : \theta = \theta_0$ against $H_a : \theta > \theta_0$.

ii. If the test in part (a) is to have $\theta_0 = 100, \theta_a = 400$, and $\alpha = \beta = 0.05$, find the appropriate sample size and critical region.

(a) On a certain day Mr.Nathan finds that a hundred rupee note is missing from his purse. He suspects that his eldest son Kumar must have taken the money. Nathan feels that if Kumar has stolen the money he should be punished. The loss table is follows,

	a_1(punish)	a_2(do not punish)
θ_1(guilty)	1	2
θ_2(innocent)	4	0

Since it was possible that one of his other sons had stolen the money. Nathan decides to base his action on the outcomes of the experiment which consists of observing whether Kumar buys a large size chocolate (Z_1), medium size chocolate (Z_2), small size chocolate'(Z_3) that day. Nathan's estimation of the probability distribution of the data is given as follows.

	Z_1	Z_2	Z_3
θ_1	0.1	0.4	0.5
θ_2	0.2	0.6	0.2

i. What are the possible strategies?

ii. Evaluate their losses and find the best strategy.

iii. If the prior distribution of θ is given as $P(\theta = \theta_1) = \frac{1}{4}$ which is the best of these strategies.

(b) The mean muscular endurance score of a random sample of 60 subjects was found to be 145 with standard deviation of 40. Construct a 95% confidence interval for the true mean. Assume the sample size to be large enough for normal approximation. What size of sample is required to estimate the mean within 5 of the true mean with a 95% confidence?