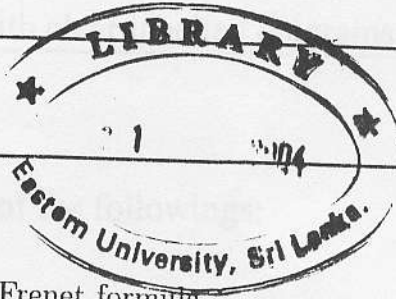


EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE 2002/03 & 2002/03(A)

SECOND SEMESTER (April/May'2004)

MT 205 - DIFFERENTIAL GEOMETRY



Answer all questions

Time: one hour

1. State and prove Serret-Frenet formula.

(a) Let  $C$  and  $C_1$  be two curves with a common principal normal line. If  $\kappa$  and  $\tau (\neq 0)$  are the curvature and torsion along  $C$  respectively, then show that there are constants  $\alpha$  and  $\gamma$  such that

$$\kappa + \gamma\tau = \frac{1}{\alpha}.$$

(b) A twisted curve  $\Gamma$  is given by the parametric equations  $X = a \tan \theta$ ,  $Y = a \cot \theta$ ,  $Z = a\sqrt{2} \log(\tan \theta)$ ,  $a > 0$ ,  $\theta$  being the parameter. If  $\kappa$  and  $\tau$  are the curvature and torsion of  $\Gamma$  at a point  $P$  respectively, then prove that

$$\kappa = |\tau| = \frac{\sqrt{2}}{4a} \sin^2 2\theta$$

2. What is meant by saying that a curve is a helix?

(a) Prove, that a space curve to be a helix if and only if  $\frac{\tau}{\kappa}$  is constant, where  $\kappa$ ,  $\tau$  are curvature and torsion of the given space curve respectively.

(b) Show that the curve  $\underline{r}(\alpha) = e^\alpha (a \cos \alpha, a \sin \alpha, b)$  is a helix, where  $a$  and  $b$  are constant.