EASTERN UNIVERSITY, SRI LANKA

SECOND EXAMINATION IN SCIENCE 2002/03 & 2002/03(A)

SECOND SEMESTER(April/May'2004)

MT 205 - DIFFERENTIAL GEOMETRY

Answer all questions Time:one hour

1. State and prove Serret-Frenet formula.

   (a) Let C and C_1 be two curves with a common principal normal line. If κ and $\tau(\neq 0)$ are the curvature and torsion along C respectively, then show that there are constants α and γ such that
   $$\kappa + \gamma\tau = \frac{1}{\alpha}.$$

   (b) A twisted curve Γ is given by the parametric equations $X = a\tan\theta, Y = a\cot\theta$, $Z = a\sqrt{2}\,\log(\tan\theta)$, $a > 0$, θ being the parameter. If κ and τ are the curvature and torsion of Γ at a point P respectively, then prove that
   $$\kappa = |\tau| = \frac{\sqrt{2}}{4a}\sin^2 2\theta$$

2. What is meant by saying that a curve is a helix?

   (a) Prove, that a space curve to be a helix if and only if $\frac{\tau}{\kappa}$ is constant, where κ, τ are curvature and torsion of the given space curve respectively.

   (b) Show that the curve $\underline{r}(\alpha) = e^{\alpha}\,(a\cos\alpha, a\sin\alpha, b)$ is a helix, where a and b are constant.