EASTERN UNIVERSITY, SRI LANKA

SPECIAL DEGREE EXAMINATION IN SCIENCE – 2009/10

FIRST SEMESTER

(May 2010)

PH 401 ELECTROMAGNETIC THEORY AND WAVES

me: 03 Hours.

nswer ALL Questions

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u may assume the following.

ector equations:

$(f\vec{A}) = f(\vec{\nabla}.\vec{A}) + \vec{A}(\vec{\nabla}f)$

$\times (f\vec{A}) = (\vec{\nabla}f) \times \vec{A} + f(\vec{\nabla} \times \vec{A})$

$(\vec{A} \times \vec{B}) = \vec{B}.(\vec{\nabla} \times \vec{A}) - \vec{A}.(\vec{\nabla} \times \vec{B})$

$\times (\vec{A} \times \vec{B}) = \vec{\nabla}(\vec{\nabla}.\vec{A}) - \nabla^2\vec{A}$

auss divergence theorem:

$$\oint_S \vec{A}.d\vec{a} = \int_V \vec{\nabla}.\vec{A}d\tau$$

okes's theorem:

$$\oint_C \vec{A}.d\vec{l} = \int_S (\vec{\nabla} \times \vec{A}).d\vec{a}$$

$= 8.85 \times 10^{-12} Fm^{-1}$ and $\mu_o = 4\pi \times 10^{-7} Hm^{-1}$

1. State Gauss Theorem in Electrostatics.

(i) A spherical charge distribution of radius R has a constant vol
charge density ρ.

(a) Calculate the electric field $E(r)$ produced by the ch
distribution when:

i. $r < R$ ii. $r = R$ iii. $r > R$

where r is the distance from the centre of the sphere.

(b) Show that the electric potential $\varphi(r)$ inside the ch
distribution when $r < R$ is:

$$\varphi(r) = \frac{\rho}{6\varepsilon_o}(3R^2 - r^2)$$

(c) Find the electrostatic energy of the sphere.

(ii) If the electric charge with volume charge density $\rho(r) = \alpha +$
distributed in a spherical shell of inner radius R_1 and outer ra
R_2,
find the expression for the electric field when,

i. $r < R_1$ ii. $R_1 < r < R_2$ iii. $r > R_2$

Assume r is the distance from the center of the spherical shell.

3.

2. Show that in a dielectric material, the bound surface charge den
σ_b and bound volume charge density ρ_b are given by $\sigma_b = \vec{P}.\vec{n}$
$\rho_b = -\vec{\nabla}.\vec{P}$
Also show that the displacement vector can be written as: $\vec{D} = \varepsilon_o\vec{E}$ +
The symbols have their usual meanings.

A spherical conducting shell has inner radius R_1 and outer radius
The volume between the spherical surfaces is filled with a med
having absolute permittivity,

$$\varepsilon(r) = \frac{\varepsilon_0}{1 + \lambda r}$$

where λ is a constant and r is the radial coordinate. A charge Q is placed on the inner surface and the outer surface is grounded.

Find:

(i) The electric field $\vec{E}(r)$ in the region $R_1 < r < R_2$

(ii) The displacement vector $\vec{D}(r)$ in the region $R_1 < r < R_2$

(iii) The potential differences between the spherical surfaces.

(iv) Polarization vector $\vec{P}(r)$ in the region $R_1 < r < R_2$

(v) Bound volume charge density.

(vi) Bound surface charge density at $r = R_1$ and $r = R_2$

A charge of $10\ \mu\ C$ exists in a spherical dielectric of relative permittivity $\varepsilon_r = 4.5$. Determine the total energy contained in the electric field outside a radial distance of 10 cm.

3. State Ampere's Circuital Law. Using this law find the magnetic field produced by a uniform current I flowing through a non magnetic hollow cylindrical conductor with inner radius a and outer radius b for the following cases:

(i) $r < a$ (ii) $a < r < b$ (iii) $r > b$

where r is the distance from the axis of the cylinder.

If the current density J varies with r according to

$$J(r) = \frac{k}{r^2}$$

show that the total current flowing in the conductor is:

$$I = 2\pi k \ln\frac{b}{a} \qquad \text{where } k \text{ a constant.}$$

4.

(i) Starting from Gauss theorem in Electrostatics derive first Maxwell's equation.

$$\vec{\nabla}.\vec{E} = \frac{\rho}{\varepsilon_0}$$

(ii) Starting from Biot-Savart law for magnetic field derive secon[d] Maxwell's equation:

$$\vec{\nabla}.\vec{B} = 0$$

(iii) Starting from Faradays law of electromagnetic induction deri[ve] third Maxwell's equation:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

(iv) Starting from the Ampere's circuital law derive the fourth Maxwell's equation:

$$\vec{\nabla} \times \vec{B} = \mu_o \left(\vec{J} + \varepsilon_o \frac{\partial \vec{E}}{\partial t} \right)$$

The symbols have their usual meanings.

6.

5. Starting from Maxwell's equation in dielectric medium where $\rho = 0$ $J = 0$, show that the electric field E satisfies the wave equation:

$$\nabla^2 \vec{E} - \varepsilon\mu \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

where ε, μ have their usual meanings and constants.

Consider a plane electric field in a dielectric medium parallel to X-ax[is] and propagating along Z-axis.

$$\vec{E} = E_o \hat{\vec{x}} \exp i(\omega t - kz)$$

where E_o, ω and k are constants.

(i) Using the Maxwell's equation show that in a dielectric medium $\vec{H}$ field is given by: $\vec{H} = \sqrt{\frac{\varepsilon}{\mu}}(\hat{z} \times \vec{E})$

(ii) A plane electric field travelling in a perfect dielectric medium is given by: $E_X(z,t) = 10\cos(2\pi \times 10^7 t - \frac{\pi}{10}z)\ V\,m^{-1}$

(a) Determine the velocity of propagation

(b) Find the associated Magnetic field intensity H if $\mu = \mu_o$

6. The electric field $\vec{E}$ in a matter satisfies the following equation,

$$\nabla^2 \vec{E} = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \varepsilon\mu \frac{\partial^2 E}{\partial t^2}$$

where the symbols have their usual meanings

(i) Consider the solution of the above wave equation as

$$\vec{E} = \vec{E}_0 e^{i(\omega t - kz)}$$

(a) Show that k and ω satisfy the dispersion relation:

$$k^2 = \omega^2 \mu\varepsilon - i\omega\mu\sigma$$

(b) Show that the skin depth z_0 in a conductor is given by

$$z_0 = \sqrt{\frac{2}{\mu\omega\sigma}}$$

(ii) When the electric wave is travelling in an ionized gas where $\varepsilon = \varepsilon_o$ and $\mu = \mu_o$

(a) Show that the dispersion relation becomes

$$k^2 = \varepsilon_o \mu_0 \omega^2 \left(1 - \frac{\omega_p^2}{\omega^2}\right)$$

and the symbols have their usual meanings.

(b) Determine the refractive index of the medium.