

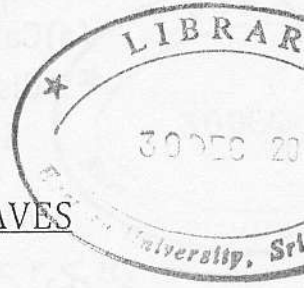
EASTERN UNIVERSITY, SRI LANKA

SPECIAL DEGREE EXAMINATION IN SCIENCE - 2009/10

FIRST SEMESTER

(May 2010)

PH 401 ELECTROMAGNETIC THEORY AND WAVES



Time: 03 Hours.

Answer ALL Questions

You may assume the following.

Vector equations:

$$\nabla(f\vec{A}) = f(\nabla \cdot \vec{A}) + \vec{A}(\nabla f)$$

$$\nabla \times (f\vec{A}) = (\nabla f) \times \vec{A} + f(\nabla \times \vec{A})$$

$$(\vec{A} \times \vec{B}) = \vec{B} \cdot (\nabla \times \vec{A}) - \vec{A} \cdot (\nabla \times \vec{B})$$

$$\nabla \times (\vec{A} \times \vec{B}) = \nabla(\vec{B} \cdot \vec{A}) - \nabla^2 \vec{A}$$

Gauss divergence theorem:

$$\oint_S \vec{A} \cdot d\vec{a} = \int_V \nabla \cdot \vec{A} d\tau$$

Stokes's theorem:

$$\oint_C \vec{A} \cdot d\vec{l} = \int_S (\nabla \times \vec{A}) \cdot d\vec{a}$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ Fm}^{-1} \text{ and } \mu_0 = 4\pi \times 10^{-7} \text{ Hm}^{-1}$$

1. State Gauss Theorem in Electrostatics.

(i) A spherical charge distribution of radius R has a constant volume charge density ρ .

(a) Calculate the electric field $E(r)$ produced by the charge distribution when:

i. $r < R$ ii. $r = R$ iii. $r > R$

where r is the distance from the centre of the sphere.

(b) Show that the electric potential $\varphi(r)$ inside the charge distribution when $r < R$ is:

$$\varphi(r) = \frac{\rho}{6\epsilon_0} (3R^2 - r^2)$$

(c) Find the electrostatic energy of the sphere.

(ii) If the electric charge with volume charge density $\rho(r) = \alpha + \beta r$ is distributed in a spherical shell of inner radius R_1 and outer radius R_2 ,

find the expression for the electric field when,

i. $r < R_1$ ii. $R_1 < r < R_2$ iii. $r > R_2$

Assume r is the distance from the center of the spherical shell.

2. Show that in a dielectric material, the bound surface charge density σ_b and bound volume charge density ρ_b are given by $\sigma_b = \vec{P} \cdot \vec{n}$
 $\rho_b = -\vec{\nabla} \cdot \vec{P}$

Also show that the displacement vector can be written as: $\vec{D} = \epsilon_0 \vec{E} + \vec{P}$
The symbols have their usual meanings.

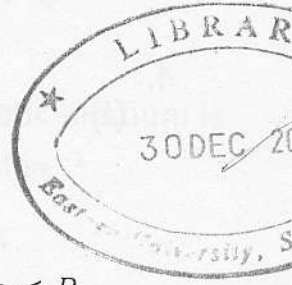
A spherical conducting shell has inner radius R_1 and outer radius R_2 . The volume between the spherical surfaces is filled with a medium having absolute permittivity ϵ .

$$\epsilon(r) = \frac{\epsilon_0}{1 + \lambda r}$$

where λ is a constant and r is the radial coordinate. A charge Q is placed on the inner surface and the outer surface is grounded.

Find:

- (i) The electric field $\vec{E}(r)$ in the region $R_1 < r < R_2$
- (ii) The displacement vector $\vec{D}(r)$ in the region $R_1 < r < R_2$
- (iii) The potential differences between the spherical surfaces.
- (iv) Polarization vector $\vec{P}(r)$ in the region $R_1 < r < R_2$
- (v) Bound volume charge density.
- (vi) Bound surface charge density at $r = R_1$ and $r = R_2$



A charge of $10 \mu C$ exists in a spherical dielectric of relative permittivity $\epsilon_r = 4.5$. Determine the total energy contained in the electric field outside a radial distance of 10 cm.

3. State Ampere's Circuital Law. Using this law find the magnetic field produced by a uniform current I flowing through a non magnetic hollow cylindrical conductor with inner radius a and outer radius b for the following cases:

- (i) $r < a$
- (ii) $a < r < b$
- (iii) $r > b$

where r is the distance from the axis of the cylinder.

If the current density J varies with r according to

$$J(r) = \frac{k}{r^2}$$

show that the total current flowing in the conductor is:

$$I = 2\pi k \ln \frac{b}{a} \quad \text{where } k \text{ a constant.}$$

4. (i) Starting from Gauss theorem in Electrostatics derive first Maxwell's equation.

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0}$$

- (ii) Starting from Biot-Savart law for magnetic field derive second Maxwell's equation:

$$\vec{\nabla} \cdot \vec{B} = 0$$

- (iii) Starting from Faradays law of electromagnetic induction derive third Maxwell's equation:

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

- (iv) Starting from the Ampere's circuital law derive the fourth Maxwell's equation:

$$\vec{\nabla} \times \vec{B} = \mu_0 \left(\vec{J} + \epsilon_0 \frac{\partial \vec{E}}{\partial t} \right)$$

The symbols have their usual meanings.

5. Starting from Maxwell's equation in dielectric medium where $\rho = 0$, $J = 0$, show that the electric field E satisfies the wave equation:

$$\nabla^2 \vec{E} - \epsilon\mu \frac{\partial^2 \vec{E}}{\partial t^2} = 0$$

where ϵ, μ have their usual meanings and constants.

Consider a plane electric field in a dielectric medium parallel to X-axis and propagating along Z-axis.

$$\vec{E} = E_0 \hat{x} \exp i(\omega t - kz)$$

where E_0, ω and k are constants.

(i) Using the Maxwell's equation show that in a dielectric medium \vec{H} field is given by: $\vec{H} = \sqrt{\frac{\epsilon}{\mu}} (\hat{z} \times \vec{E})$

(ii) A plane electric field travelling in a perfect dielectric medium is given by: $E_x(z, t) = 10 \cos(2\pi \times 10^7 t - \frac{\pi}{10} z) \text{ V m}^{-1}$

(a) Determine the velocity of propagation

(b) Find the associated Magnetic field intensity H if $\mu = \mu_0$

6. The electric field \vec{E} in a matter satisfies the following equation,

$$\nabla^2 \vec{E} = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \epsilon\mu \frac{\partial^2 \vec{E}}{\partial t^2}$$

where the symbols have their usual meanings

(i) Consider the solution of the above wave equation as

$$\vec{E} = \vec{E}_0 e^{i(\omega t - kz)}$$

(a) Show that k and ω satisfy the dispersion relation:

$$k^2 = \omega^2 \mu \epsilon - i \omega \mu \sigma$$

(b) Show that the skin depth z_0 in a conductor is given by

$$z_0 = \sqrt{\frac{2}{\mu \omega \sigma}}$$

(ii) When the electric wave is travelling in an ionized gas where $\epsilon = \epsilon_0$ and $\mu = \mu_0$

(a) Show that the dispersion relation becomes

$$k^2 = \epsilon_0 \mu_0 \omega^2 \left(1 - \frac{\omega_p^2}{\omega^2} \right)$$

and the symbols have their usual meanings.

(b) Determine the refractive index of the medium.

